Chapter 11

Fission

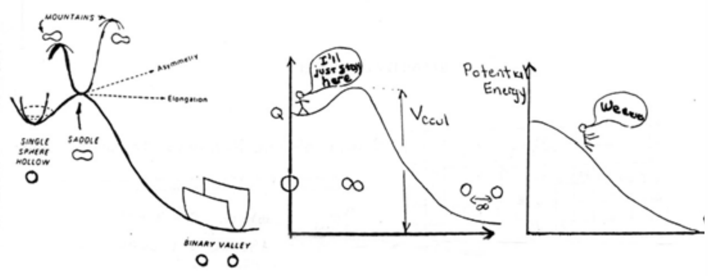

Figure 11.1: Schematic view of a) fission metastability, the saddle point and b) middle and c) end of the periodic table.

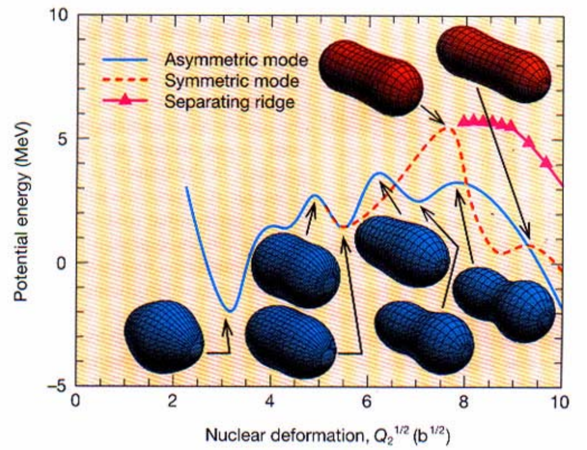

Figure 11.2: Taken from: P. Moller, D.G. Madland, A. J. Sierk, and A. Iwamoto, Nature **409**, 785 (2001).

All nuclei removed from the Fe region are meta stable. However metastable nuclei can live forever as nuclei are well **isolated systems**. The sites for nuclear reactions are limited because the kinetic energies typical of the common temperatures in the universe are so low that nuclei are preventing from contacting one anther. This isolation is offered by either the atomic hard-cores (e-e repulsion), or (at higher energy) the nuclear Coulomb barrier.

Alpha decay becomes energetically possible when the binding energy per particle of the parent is less than that of the daughter and α-particle. Even so, lifetimes can be long as the Coulomb barrier must be penetrated. Similarly nuclei become unstable w.r.t. to fission when the parent's binding energy (per particle) is less than that of two approximately half-sized nuclei. If this thermodynamic condition determined what we call "stable", we would only have about 50 elements. The periodic table extends significantly further as the upper half (of the table) is "*kinetically*" trapped behind a huge (fission) barrier B_f. The periodic table ends when the barrier gets close to vanishing.

Typically one represents the progress of a heavy nucleus changing from a "**mononucleus**" into two separated nuclei as a process proceeding through a transition region which topologically looks like a mountain pass, or "**saddle point**". However (as the case with chemical reactions) a one-dimensional **potential-energy surface** (PES) picture see Fig. 11.1 is misleading. In truth, the PES is very complicated and requires at least 6 shape degrees of freedom to describe the relevant dynamical shapes between the initial ground state (through the critical barrier saddle-point shape) to the neck breaking or **scission shapes**, see Fig. 11.2.

11.1 Mass distributions

One of the most revealing observations concerning (thermal neutron) fission of actinides is that the two fragments differ in size.[1] One can understand why actinide

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[1] The most complete compilation of fission yields are those complied by Art Wahl of WU. The complete list of yields is found in, A.C. Wahl, Atomic and Nuclear Data Tables **39**,1, (1988). The simple list of "cumulative" isobaric yields, for slow n induced fission of ^{233,235}U and ^{239}Pu are shown in the most south-easterly

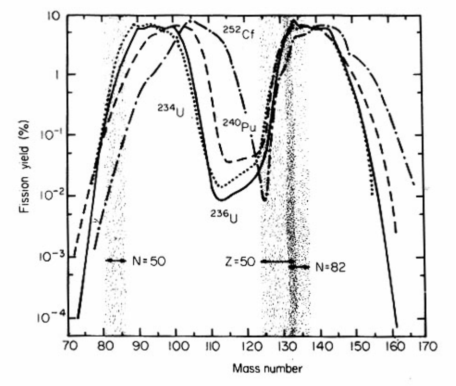

Figure 11.3: Neutron induced fission yields of ^{235,8}U and ^{239}Pu and the spontaneous fission of ^{252}Cf.

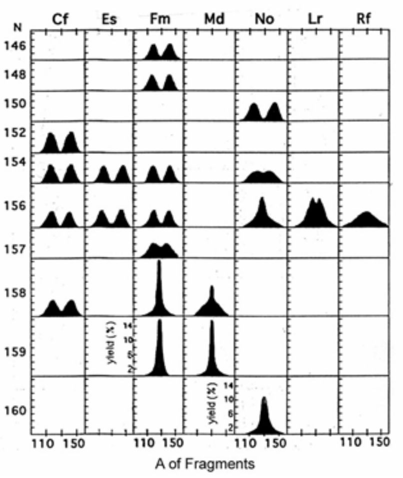

Figure 11.4: Schematic fission yields at various places in the periodic table.

fission is asymmetric after observing how the mass distributions change with increasing parent mass, see Fig. 11.3. The upper mass peak remains fixed at about $A \sim 140$ while the light peak moves up (with increasing mass of the parent), as required to conserve mass. The upper mass peak is fixed by the extra stability of offered this fragment by the N=82 shell closure. (The very lightest fragments in the heavy mass peak are also influenced by the Z=50 closure.) The influence of the N=82 shell closure is also seen in the fragment excitation energy, which is less for the heavy fragment than for the light one.

With increasing mass of $_{100}$Fm isotopes, the two peaks merge as both fragments can find stabilization from the Z=50 shell gap and get close to the N=82 gap. However, the mass distributions also reveal that in certain isotopes there is a competition between two fission channels. One yielding a peak near symmetry and, the other, the standard doublet of asymmetric peaks. The lesson from this is that there are multiple "**channels**" in the multi-dimensional potential-energy surface taking the nucleus from the equilibrium shape to saddle-point shapes to scission shapes. Individual events (fissions) can take one route or another, and the different routes lead to different mass distributions, energy distributions and number of promptly released neutrons.

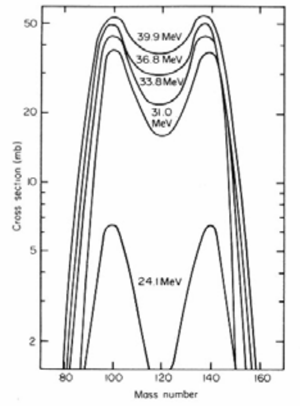

Figure 11.5: ^{4}He indiced fission of ^{238}U.

When fission occurs from highly excited nuclei, the mass distribution is symmetric. This is what one would expect from the liquid drop model for the process. The shell structure which drives the mass distributions asymmetric, is only relevant when the parent is near its ground state (nucleons are, for the most part, in the lowest single-particle levels.)

11.2 Gross Energetics

The typical actinide fission releases 2-3 neutrons along with the two fission fragments. (These free neutrons are key to understanding how fission can be self-sustaining.) Even with the emission of these neutrons, the fission fragments are neutron rich and thus they β-decay back to stability. Thus when one describes fission, one must consider:

1. the initial breakup of a heavy nucleus into two fragments,
2. the evaporation of a few free neutrons from the "hot" primary fission fragments,

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tip of each isobaric chain on the Chart of the Nuclides.

3. the β decay of the "cold" fission fragments,
4. the γ decay of the excited states fed by the β decay,
5. the capture of the evaporated neutrons,
6. and the decays of the nuclei produced by the capture (further fission or beta decay.)

   This matrix of possibilities requires computer modeling and, in fact, was the impetus for building some of the first digital computers and the creation of Monte-Carlo simulation methods

   Lets consider the energetics of a typical fission event.
   $n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba^{*} + {}^{95}_{36}Kr^{**}$
   $\rightarrow {}^{140}_{56}Ba + {}^{93}_{36}Kr + 3n + Q$
   $Q = +42.4 - [-84.9 - 65.6 + 3(8.071)] = 170\ MeV$
   $Q = T_H + T_L + E^*_H + E^*_L$.

The Q-value approaches 0.8 MeV per particle for actinides. (This is just the difference in binding energy between the upper end of the periodic table and the better bound nuclei with about half the charge and mass.) This energy is distributed to the kinetic energy of the fragments and their internal excitation energy. About 90% of the energy is kinetic with $KE_L > KE_H$. These kinetic energies are about what one would expect from two charged objects separating at a distance somewhat larger than that of two touching spheres and repelling one another as they "fall" down the Coulomb potential. The internal excitation energy is spent primarily by emitting neutrons. This "evaporative" cooling process, which accounts for almost all[2] of the 2-3 neutrons emitted during actinide fission, proceeds until the internal energy of each fragment is less than the neutron binding energy. Photon emission removes the remainder of the energy.

The kinetic energy partition is a result of momentum conservation $p_L = p_H$, thus

$\frac{(p_L)^2}{2M_L} = \frac{(p_H)^2}{2M_L}$

$KE_L = \frac{(p_H)^2}{2M_L}\frac{M_H}{M_H} = KE_H\frac{M_H}{M_L}$.

The total fission fragment kinetic energies (TKE) increases (decreases) with parent Z (A), see Fig. 11.6. This is to be expected from Coulomb logic. On a per fragment basis, the kinetic energies are about 0.8 MeV/amu which is just above the region where nuclear stopping becomes more important than ionization. (This initial KE, is in the "Bragg-peak" region.) Fission fragments slow down (initially by ionization and then by nuclear stopping) and are completely stopped in a few μm of any condensed phase or about 1 cm of air. However, the associated radiation (γ's and neutrons are characterized by exponential absorption, the latter with small absorption coefficients.)

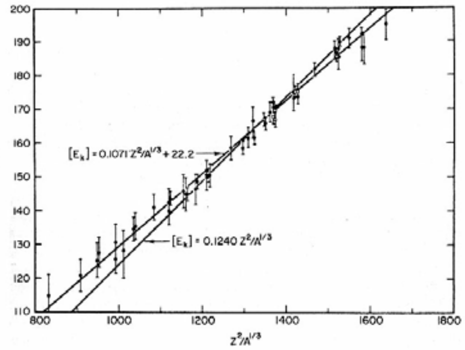

Figure 11.6: Fission fragment TKE as a function of $Z^2/A^{1/3}$.

The internal excitation energies, $E^*_L > E^*_H$, are first spent first on neutron emission (as the neutrons are free, they "cost" their binding energy to be emitted) and then on photon emission. The asymmetric partition is a consequence of the heavy fragment being more spherical at the scission (breaking) point. To amplify on one point made above, the free neutrons are almost completely from the decay of the fragments rather than being produced at scission. This is important in that the neutrons are kinematically focused along the direction of the two fragments. (This focusing is not strong but does produce about a 2/1 ratio of the neutron emission probability along/perpendicular to the fission axis.)

11.3 Beta decay

The line of β-stability is given approximately by $A \sim 1.753 \cdot Z + 0.0865 \cdot Z^{1.5}$. As a result of the "super" linear excess of neutrons relative to protons with increasing Z, fission fragments are neutron rich and thus β^- unstable.[3] The β-decay (and attendant γ-decay) are irrevocable parts of the total fission process. Fission thus produces neutrons, β 's , γ's as well as provides a means to produce neutron rich activities in the middle of the periodic table.

Many of the radionuclides used in applied nuclear science are fission products (e.g. ^{137}Cs). Several companies and government agencies, exist to separate out individual activities and sell them. On the other hand, these beta activities are one of the two major radioactive

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[2] A small (< 1%) of the neutrons come AFTER β-decay. These "β-delayed" neutrons arise from the decay of the most neutron rich fission fragments and are central to understanding how nuclear reactors are conrolled.

[3] $(Z/A)_{^{238}U}$ =0.386 and thus a fragment with A=140 with the same ratio would have Z=54. The heaviest stable $_{54}$Xe is ^{136}Xe.

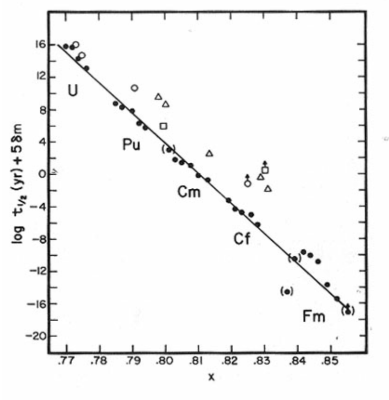

Figure 11.7: Fission half-life as a function of fissility. The solid dots are e-e, open dots e-o, open trianges o-e and open squares o-o.

waste streams produced in nuclear power reactors.[4]

11.4 Fissility

It is particularly useful in view nuclear fission as a balance between the cohesive (attractive) nuclear force and the disruptive Coulomb force. Fission occurs when the latter "wins". A parameter which represents this interaction balance is the fissility χ.[5]

$$\begin{aligned}\chi &\sim (\frac{disrutive}{cohesive}) = \frac{Coul}{2 \cdot Surf} \\ &= \frac{0.717 \cdot Z^2/A^{1/3}}{2 \cdot 18.6 \cdot A^{2/3}(1 - 1.8 \cdot I)} = \frac{Z^2}{51.6 \cdot A^3(1 - 1.8 \cdot I)}\end{aligned}$$

As χ increases the spontaneous fission half-life decreases, Fig. 11.7. When $\chi = 1$, the fission barrier disappears and the nucleus becomes unstable wrt to deformation and thus fission occurs instantaneously. One should note here that fissility not only increases with increasing Z but with decreasing A. (Packing a fixed number of protons into a smaller box increases the Coulomb instability.) These trends are the same as occur in α-decay. (Recall that some n deficient rare earths are spontaneous α-emitters.)

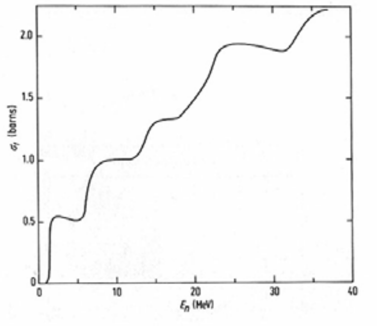

Figure 11.8: Steps in multichance fission. Each step corresponds to fission after another neutron is emitted.

11.5 Multichance fission

If a high energy particle is shot at a heavy nucleus, fission can occur directly or after a neutron is emitted. We will discuss the compound nucleus in another chapter, but for now I want you to understand the following multistep logic. (This is, what mathematicians would call a Markov chain.) Say we create a ^{239}U nucleus with $E^* = 50$ MeV. The excited compound nucleus can fission or "evaporate" a particle. (Emission of charged particles is inhibited by the Coulomb barrier, so this evaporation is almost always a neutron.) If a neutron is emitted, the excitation is reduced by the binding energy of the neutron (about 7 MeV) plus whatever kinetic energy the neutron has (about 1 additional MeV). This "first chance daughter" can itself fission or emit a neutron. This chain can continue until all the energy is spent. As a result of this chain, the integrated (summed) fission probability increases in a step-like fashion with increasing initial excitation energy. The stair-step dependence on excitation energy, see Fig. 11.8, is the result of the discrete cost of evaporating neutrons. (The nucleus must expend - one neutron binding energy worth of energy - to emit each neutron.)

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[4] The other is actinides produced by n capture on U. Crudely speaking, the bulk of the radioactivity at short times (< few hundred years) from reactors is the β^- decays of the fission fragments while the activity remaining at long times (> 1000 years) is the actinides. Separation of the fission product waste from the actinides 1) produces a waste stream which will substantially decay in a few hundred years, 2) creates an actinide stream which can be used to fuel other reactors but also 3) enhances the potential for proliferation.

[5] The factor of 2 which multiplies the surface denominator term, results when one considers the change in these interactions with deformation (in a Taylor) expansion.