THE SPIN SPECTROMETER: DESIGN, INSTRUMENTATION AND RESPONSE
CHARACTERISTICS OF A 4\pi \gamma-RAY MULTIDETECTOR SYSTEM

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Construction of a new type of spectrometer for the investigation of nuclear structure at high angular momentum and of the mechanisms of heavy-ion induced reactions is reported. The spectrometer consists of 72 NaI(Tl) detectors closely packed in a 4\pi arrangement. It is capable of recording on an event-by-event basis for each detector that fires (1) the identification number, (2) the pulse height, (3) the time of flight, and (4) the pulse width. From these data we can obtain for each event: the \gamma-ray multiplicity, the total pulse height and thus the total \gamma-ray energy, the neutron multiplicity, the angular correlations of the emitted \gamma-rays, and the time relationships between the \gamma-rays in a cascade. Pulse height spectra of the \gamma-rays and of the neutrons associated with the above parameters can also be obtained. The geometric arrangement, the performance of the individual detectors, the associated electronics, the data acquisition system, the response of the entire system to incident \gamma-cascades of given multiplicity and total energy (including the effect of coincidence summing and detector-to-detector scattering), and the response to incident neutrons are discussed for various approximate and realistic inputs. The unfolding of populations of total pulse height and coincidence fold to yield populations in excitation energy and \gamma-ray multiplicity is described. The unfolding of pulse-height spectra observed in individual detectors to yield \gamma-ray energy spectra is also discussed.

I. Introduction

Detection of the electromagnetic radiation emitted in heavy-ion induced reactions provides a good tool for the investigation of both nuclear structure at very high angular momentum and reaction mechanisms. In such investigations, experiments are becoming increasingly complex because high \gamma-ray multiplicities are usually involved in heavy-ion reactions.

Measurements of the \gamma-ray multiplicity (M) have been made by utilizing high order coincidence techniques [1-9]. With the application of up to fourteen NaI detectors useful information about the angular momen-

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provide a connection between the observed pulse height and total \( \gamma \)-ray energy on an event-by-event basis, they provide only a crude estimate of \( M \) and they suffer from three serious drawbacks: (1) The small target-to-detector distance makes it very difficult to distinguish the pulses due to neutrons from those due to \( \gamma \)-rays. (2) The total-energy efficiency of the sum spectrometers built thus far is typically 60–65\%. (3) The correlated total energy and the associated multiplicity cannot be deduced simultaneously.

The above points emphasize the need for a new type of instrument capable of measuring simultaneously the \( \gamma \)-ray multiplicity and the total \( \gamma \)-ray energy. Such an instrument was proposed recently by Sarantites et al. [15]. The main criteria for the design of this type of spectrometer are: the number of the detector elements in the sphere should be at least twice the maximum multiplicity to be measured; the detector shell must subtend a solid angle of nearly \( 4\pi \) and be sufficiently thick for nearly complete absorption of the \( \gamma \)-rays involved; and the inner diameter of the shell should be large enough for adequate separation of \( \gamma \)-rays from neutrons by time-of-flight (TOF) and for accommodating a large reaction chamber, essential to a versatile instrument.

This kind of spectrometer can be used in a large variety of experiments and can provide unique information about the properties of nuclei at very high angular momentum and about the mechanisms of heavy-ion induced reactions. The experimental quantities that can be measured by such a device on an event-by-event basis are: the \( \gamma \)-ray coincidence fold and the associated total pulse height, the pulse heights of individual \( \gamma \)-ray transitions, the neutron multiplicity, the angular correlations among the \( \gamma \)-rays in the cascade, the spin direction of the initial state in the \( \gamma \)-cascade, and the time and energy correlations among the various groups of \( \gamma \)-rays in each cascade.

In this paper we describe the performance of such a spectrometer built at Washington University and installed at the Holifield Heavy-Ion Research Facility (HHIRF) at the Oak Ridge National Laboratory. This spectrometer, known as the spin spectrometer, has been installed in a beam-line at the HHIRF, which provides heavy-ion beams from the Oak Ridge Isochronous Cyclotron (ORIC), the 25-MV tandem accelerator, or from coupled operation of the 25 MV tandem injected into the ORIC.

The specific design and geometry of this apparatus are described in section 2 and the performance of the individual detectors is given in section 3. The electronics and data acquisition systems are discussed in section 4. The methods of calibration of the spectrometer are described in section 5. Finally, neutron identification and the response of the spectrometer to neutrons are discussed in section 6.

2. Description

2.1. Geometry

The spin spectrometer consists of an array of up to 72 closely packed NaI detectors approximating a hollow sphere with inner radius 178 mm and shell thickness 178 mm. Each detector is a tapered prism that can be removed radially. The cross section of twelve of the detectors is a regular pentagon. Each of the pentagonal detectors is surrounded by five identical detectors of irregular hexagonal cross section. The geometry of the shell surface of the spectrometer is illustrated by the polyhedron shown in fig. 1. The polyhedron formed by the front faces of the detectors is shown in fig. 1a where the pentagonal surfaces are marked in black. This polyhedron may be thought to originate from the pentagonal hexecontahedron (PHH) consisting of 60 irregular pentagons (fig. 1b) by cutting off the sharp corners of each of the 60 faces and replacing the 12 five-fold vertices by regular pentagons. Thus the truncated PHH has 60 hexagonal faces and 12 pentagonal ones. The cutoff line was chosen so that the area of each pentagon is 1.016 times that of each hexagon in order to set the solid angles of all the detectors equal. Each pentagonal detector is located directly opposite another pentagonal detector. One such pair has been removed from the spectrometer to accommodate the beam entry and exit pipes which are welded to the inner spherical reaction chamber. For experiments in which the beam is stopped in the chamber, the hemisphere with the exit pipe can be replaced by a plain hemisphere to allow reinstallation of the missing NaI detector or use of an external detector at 0°.

Fig. 1. (a) Geometry of the spin spectrometer based on the truncated pentagonal hexecontahedron. The black areas represent the twelve regular pentagons. (b) The Archimedean pentagonal hexecontahedron. The polyhedron of (a) originates from truncation of the sharp corners of the twelve groups of five pentagons of (b).
the center of the largest circle inscribed into the detector face. The latter is also the center of the largest cyclindrical external detector that can be inserted in the spectrometer in place of a hexagonal element. For the pentagonal detectors these two centers coincide.

The effective solid angle of each NaI crystal is 1.344\% (96.8\%/72) of 4\pi. The 3.2\% loss is due to the wall thickness (1 mm) of the aluminum cans housing the NaI crystals and the internal light-reflecting material (0.25 mm paper) inside the cans. There is no shielding between neighboring detectors. Hemispheres of 0.5-mm thick Cu are available for use as absorbers inside the reaction chamber to reduce the target X-rays.

2.2. Triggering of the spectrometer

Many kinds of nuclear events may occur under heavy-ion bombardment and the spectrometer has a very high efficiency for detecting the radiation produced in any of them. Each experiment is therefore set up with auxiliary detectors to select the events of interest to a particular investigation, and data from the spectrometer for a given event are recorded only if a valid trigger has been supplied by the auxiliary system. Triggering by the spectrometer itself is sometimes also required.

The reaction chamber (see below) permits various detector systems to be used inside the spectrometer, for example, small \( \Delta E \times E \) silicon telescopes, NaI charged-particle detectors, or avalanche counters. External detectors such as Ge counters or neutron time-of-flight counters may also be used provided that NaI detectors are removed from the spectrometer to permit an unobstructed view of the target at the center of the sphere.

2.3. Reaction chamber

The reaction chamber consists of Al hemispheres 3-mm thick with an inner diameter of 32 cm. It has four access ports, one of 6-cm diameter for the target holder and three of 8-cm diameter. The latter are coaxial with the NaI detectors at (\( \theta, \phi \)) angles of (87.6\°, -102.6\°), (116.6\°, -72\°) and 138.6\°, 86.2\°). Re-entrant covers for these ports have been provided such that a Ge(Li) detector can be placed as close as 7.5 cm from the target. Any NaI detector in the spectrometer can be replaced by a Ge(Li) detector at a distance greater than 16.3 cm (the outer radius of the reaction chamber) or at closer distances if a different chamber is used. The reaction chamber has been equipped with removable internal hardware for supporting particle detectors. The particle-detector holders are mounted on supports which allow for positioning in five-degree steps on both sides of the beam. These supports are sections of circular rings that may be clamped at \( \phi = 0\°, 90\° \) and 180\°. Coaxial cables for internal detectors are brought out through an annular space within the entrance pipe. In
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An experiment performed recently, 11 silicon surface-barrier telescopes (two of them triple telescopes) and two evaporation residue detectors were placed inside the chamber, with a total of 26 particle detectors.

3. Performance of the NaI detectors

Each detector consists of Poisyscin NaI(Tl) [16] coupled via a plano-concave glass window to an RCA 4522 photomultiplier tube. The voltage divider in the tube base (fig. 3) draws a current of 1.8 mA at -1900 V, a typical operating voltage. This voltage provides good timing resolution and an anode signal of ~80 mV across 50 Ω for $^{60}$Co (1332 keV). The dividers allow for the adjustment of the potential of the focus grid in the phototube and the potential between the last dynode and the anode.

Linearity and counting-rate stability tests were made on several detectors as a function of the tube-base adjustments. For linearity tests, a mixture of $\gamma$-ray sources was used: $^{57}$Co, 122 keV; $^{22}$Na, 511 and 1275 keV; $^{88}$Y, 898 and 1836 keV; $^{244}$Pu + $^{209}$Be, 4438 and 3927 keV (single-escape peak); and $^{244}$Cm + $^{13}$C, 6130 and 5619 keV (single-escape peak). Higher energies were simulated by increasing the phototube voltage and attenuating the anode output so as to present the analysis system with the same pulse amplitudes. A given nonlinearity from high-energy $\gamma$-rays could be produced from low-energy $\gamma$-rays with the tube operating at higher than normal voltage by signals attenuated to equal height. In

![Fig. 3. Schematic diagram of the voltage divider chain in the phototube base.](image-url)
this way the linearity was measured up to equivalent energies of ~50 MeV. The counting-rate stability was measured with $^{60}$Co sources of different strength.

The best counting-rate stability with the present voltage dividers is achieved when the tube base controls are adjusted for nearly the maximum pulse-height output. With these settings, however, the potential between the 14th dynode and the anode is typically only 1% of the total voltage and the phototubes are nonlinear for $E_r \gg 3$ MeV. When this potentiometer is not at maximum (14th dynode–anode potential ~10% of the total), the detectors give a linear response up to ~50 MeV, but the counting-rate stability is poorer by a factor of two or three. Thus, depending on the application, the instrument can be optimized for linearity or stability. In the experiments run thus far, the latter choice has been made and the pulse height were corrected for the nonlinearity determined during the post-run calibrations with $\gamma$-ray sources.

The counting-rate stability of a typical detector is illustrated in fig. 4 for a wide range of operating voltages. Below 1700 V there is little or no peak shift from 10000 to ~50000 c/s with $^{60}$Co for a threshold of ~100 keV. For higher voltages a linear shift of 0.7% per 10000 c/s is observed. With the voltage dividers adjusted for best stability at 1332 keV, the upper limit for 1% peak shift relative to 10000 c/s lies in the range 35000–90000 c/s, varying with different phototubes. With many tubes, an additional count-rate shift of 1–3% occurs below about 5000 c/s. During the experiments run so far the count-rate has been in the range 8000–20000 c/s per detector.

Neutrons are distinguished from $\gamma$-rays by their time of flight. The higher intensity and average velocity of neutrons at forward angles requires that the detectors with better timing performance be placed in forward positions. Before assembling the spectrometer, the timing performance of each detector was measured for $^{60}$Co coincidences between the NaI and an NE-213 liquid scintillator. With the threshold set at 80 keV, the average fwhm for the NaI detectors was 2.1 ns and the average fwtm was 4.7 ns. The fwhm varied between 1.7 and 2.5 ns and the fwtm between 3.8 and 5.4 ns. For the detector placement actually adopted during assembly of the spectrometer, fig. 5b shows the time resolution averaged over each group of five detectors having the same $\theta$. All detectors in the forward hemisphere have fwtm better than 4.7 ns. Reducing the high voltage by 100 V increases the fwtm by about 0.2 ns, but higher voltages do not improve the timing performance significantly. The energy resolution of the NaI detectors varies between 7.7 and 9.2% for the 662 keV $\gamma$-ray from $^{137}$Cs and between 5.6 and 6.7% for the 1332 keV $\gamma$-rays from $^{60}$Co; the average values are 8.6 and 6.3% for the 662 and 1332 keV lines, respectively.

The average energy resolution of the same detector groups is shown in fig. 5a for the 662 and 1332 keV $\gamma$-rays. A correlation between timing and energy resolution is clearly seen: detectors that have good timing tend also to show good energy resolution. The energy resolution is not sensitive to the high voltage, but it deteriorates somewhat at very high counting rates. For counting rates below 50000 c/s, less then 0.1% decrease in energy resolution at 662 keV was observed. At 75000 c/s the average energy resolution for 662 keV was 9.9%. The pulse height for the 662 keV line from $^{137}$Cs was found to decrease by 0.3% per °C over the ambient temperature range of 18–24°C. The maximum gain shift over a period of two weeks that was not correlated with the temperature was 1%.

The pulse-height variation along the detector length as measured with a collimated $^{137}$Cs source was less than 3% on the average.
4. Electronics and data acquisition

4.1. Hardware

A schematic diagram for the electronics of the spectrometer [17] is shown in fig. 6. The photomultiplier anode signal is first amplified by a fast gain-10 amplifier [18]. One of the two amplifier outputs is shaped to an approximately triangular pulse (~100 ns FWHM and ~200 ns FWHM), attenuated, delayed by ~300 ns, and then fed into a charge-integrating CAMAC ADC [19]. The second output of the ×16 amplifier is used to establish the timing of the anode pulses by means of two constant-fraction discriminators (CFD) of special design that examine the rising and falling portions of the NaI pulses.

The front-edge CFD establishes the basic timing for each pulse. The time difference between the front-edge and rear-edge triggers provides information about possible pile up or coincidence summing between a neutron and a γ-ray pulse in the same detector. The outputs of the two CFDs generate a fixed amplitude pulse with a variable width, which is proportional to this time difference. These variable-width pulses are digitized by a charge-integrating CAMAC ADC [19]. A second pulse occurring at a time ~8–240 ns after the first pulse can be sensed with this technique. The output of the front-edge CFD is delayed, stops a time-to-digital converter (TDC) [20], and sets a detector identification bit in a gated latch. A 72-channel digital multiplicity summer (fast fold select in fig 6) provides the coincidence fold for a possible fast hardware decision as to whether or not to record the event.

The entire system must be triggered by an external signal satisfying the criteria for an acceptable event. Suitable strobe signals with appropriate delay and width adjustments are generated to gate all the ADCs, start the TDCs, and gate the 72 latches for the NaI detectors. The information from any auxiliary detectors is entered via a user's module, which includes 16 independent inputs with adjustable delays. Internal signals are used to set appropriate bits in a user's gated latch and external signals are generated to stop the user's TDCs. All spectrometer ADCs and TDCs as well as ADCs and TDCs from the auxiliary detectors are interfaced in a CAMAC system. The six 16-bit words from the gated latches (72 bits for the NaI pattern, 8 bits for the fold and 16 bits for the user's gated latch) are accessed through a CAMAC input register.

Data acquisition is managed by the Event Handler [21], a programmable CAMAC-based controller which selects events that meet the user's criteria, places them in a suitable format, and transmits them to the on-line computer for permanent recording on magnetic tape. The computer program may apply further selection, buffering, and formatting before the events are recorded, and it provides for on-line monitoring of the data. All CAMAC ADCs, TDCs and registers including those for the auxiliary detectors are interfaced via the same Event Handler. The time required for digitizing the signals, transmitting them to the computer, and recording on magnetic tape limits the data acquisition to 3000 events per second for events with average multiplicity ~20 [17].

4.2. Setup and monitoring

The operating characteristics of all NaI detectors are stored in a disk file. The setup and data-acquisition programs use this file to obtain information about detector parameters and CAMAC addresses for the electronics associated with each detector. This file can be updated by setup programs or edited by the user to suit his experimental requirements.

All detectors are operated with the same gain, typically 5.0 keV/channel, which provides a 10 MeV range with the 2048-channel ADCs. The gains are set by a computer program that adjusts the phototube voltage on the basis of spectra for a γ-ray source with two well-separated lines (for example, 88Y). A peak search routine determines the location of the selected lines and the voltages required to give the desired gain are calculated. The algorithm for the voltage change is based on the observation that a 100 V change in voltage gives a factor of 1.8 change in pulse height. The phototube voltages, furnished by CAMAC-interfaced power supplies [22], are then set to these values by the computer and the process is iterated. Adjustment to ~0.5 V (the smallest change possible with the high-voltage supplies) results in gain variations of less than 1% between detectors. The final adjustment of gains is done in software during playback of the event tapes on the basis of post-run calibrations described later.
Checking and setting up the instrument can be accomplished with any of three computers, an LSI-11 microprocessor-based system, an SEL-840 computer, or a Perkin–Elmer (PE) 3220 computer. Since the PE computer has the fastest central processor unit and the tape units of highest speed and density, it is ordinarily the one chosen for writing event tapes. The data stream from the Event Handler is sent via a first-in-first-out buffer (FIFO) directly to one of the three computers, or it can be sent through a FIFO event sampler, which sends the complete data stream to the PE computer while allowing one of the other computers to sample the event stream at its maximum capability without interfering with the writing of event tapes (see fig 6). This feature provides flexibility for monitoring the data during acquisition. The LSI-11 can monitor either one or two parameters at a time from the complete configuration of the experiment and loop through all the detectors sequentially. A live display of the data from any two parameters can be created with 256 × 512 channel resolution on a storage screen or the data can be stored in memory as a 140 × 140 channel array. The other computers have larger storage areas and can acquire data from all the detectors simultaneously. The acquisition programs for the PE computer provide for monitoring by creating in core or on disk one- or two-parameter arrays from all the 72 × 3 parameters of the spectrometer and those of the auxiliary detectors. Since the tape writing task is given the highest priority, the percentage of the total data monitored varies with the data flow rate and the number of one- or two-parameter arrays that are generated.

The Event Handler continuously monitors the status of the high-voltage power supplies. If it detects an error condition it halts data acquisition and generates an audio alarm to alert the experimenters.

The time scale is set approximately the same for each detector by manual adjustment of each TDC unit. The thresholds for the constant-fraction discriminators are also adjusted manually. The discriminators were adjusted to minimize the dependence of timing on pulse height ("walk"). In the experiments run thus far, the observed walk has been small enough to be ignored. The data analysis programs allow for event-by-event correction of walk if necessary.

5. Calibration methods

It is of fundamental importance to predict the outcome of an event consisting of \(\{E_i\}_{i=1}^M\) γ-rays. The response of the spectrometer to this input is determined by several physical effects that depend on the geometry of the spectrometer and on the properties of the γ-cascades and other emitted penetrating radiations. The most significant effects are (i) incomplete detection of γ-rays, (ii) detector-to-detector scattering, (iii) coincidence summing in each detector element, and (iv) pulses originating from other sources, e.g., neutrons and/or high-energy protons. The first three of these are connected to the detection of one or more γ-rays by one or more detectors in the spectrometer and must be taken into account quantitatively in the calibrations. Some of the γ-rays that escape detection do so through the holes in the spectrometer shell or are absorbed in the cans without encountering the scintillator. Furthermore, the shell thickness is not sufficient to absorb completely all the high-energy γ-rays or all of the secondary radiation produced in an initial interaction. The scattering from one detector to its neighbor is appreciable due to the absence of shielding between detectors, necessary to maximize the total efficiency. The coincidence summing arises from the finite probability that more than one γ-ray from a given event may strike the same detector, and is significant when the γ-ray multiplicity is high.

In order to achieve better resolution in multiplicity and total energy for the spectrometer, it is of great importance to discriminate against pulses which are not due to γ-rays. Most of the neutrons can be identified by their time of flight. It is also possible to distinguish between pulses arising from γ-neutron coincidence summing and those due to γ-rays by using the pulse-width discrimination (see section 4). The effect of neutrons that cannot be distinguished by these methods has to be taken into account in the data analysis. This can be done either by measuring the effective neutron response and scattering and including these in the response functions, as described in ref. 15, or by applying the correction on event-by-event basis using an "equal-probability separation line" for neutrons and γ-rays in a map of time of flight vs. pulse height. This method is discussed in section 6.

We divide the discussion of the spectrometer response into two parts. First, we give the probability \(P_n(EM, Hk)\) for a system of \(N\) detectors to record a total pulse height \(H\) and fold \(k\) for an input event of total energy \(E\) and multiplicity \(M\), where \(H = \Sigma_{i=1}^N h_i\) is the sum of pulse heights above some threshold \(h_i\), and \(E = \Sigma_{i=1}^M E_i\) is the sum over the γ-ray energies in the cascade. These probabilities are needed for transforming the observed distributions \(Q_x(H, k)\) for any selected class of events \(x\) to the populations \(R_x(EM, M)\), or for reporting the distribution \(\Delta R_x(EM, M)\) of events that lie between \(H\) and \(H + \delta H\) and have a given \(k\). Second, we shall give the probability \(p_n(EM, E, h)\) to register a pulse \(h\) in the detector element \(n\) from an input γ-ray spectrum \(E_n + \{E_i\}_{i=1}^M\) with \(E = (\Sigma_{i=1}^M E_i) + E_n\). These probabilities \(p_n(EM, E, h)\) are needed to unfold the pulse-height spectrum observed in a given NaI detector in order to obtain the associated energy spectrum.

Ignoring the angular correlation effects, analytic expressions for \(P_n(EM, Hk)\) have been derived in ref. 15.
Further insight into the structure of $P_N(EM, HK)$ has been given in section 2.3.3 of ref. 15 by expanding it in terms of its different moments. In principle, it is possible to obtain $P_N(EM, HK)$ from eq. (18a) of ref. 15 using experimentally determined single-photon responses, which appear in parentheses in eq. (14) of ref. 15. The complexity of eq. (18a) of ref. 15 makes routine evaluation of $P_N(EM, HK)$ impractical. Expansion in terms of the first few moments is very convenient for rapid evaluation of the projections of $P_N(EM, HK)$ on either the $H$ or $K$ axis. However, it cannot be used for the full characterization of the exact bivariate distribution $P_N(EM, HK)$ because that requires, in addition to the first three moments in $H$ and in $K$ evaluated in ref. 15, cross correlation terms of higher order, i.e. $\langle H^2 k \rangle$, $\langle H K^2 \rangle$, etc., which are also difficult to obtain analytically.

In this paper we present an experimental method for obtaining a complete characterization of $P_N(EM, HK)$ using suitable $\gamma$-ray sources.

We begin with the simple case of an input cascade of M $\gamma$-rays with equal energy $E_0 = E/M$. We place in the spectrometer a source emitting two $\gamma$-rays in cascade with known branching ratio as close to unity as possible, (e.g. $^{60}$Co with two $\gamma$-rays, 1173 and 1332 keV). A Ge(Li) detector is used to trigger the data acquisition system, and the response of the entire spectrometer is recorded on magnetic tape by writing in an event-by-event mode all the pulse heights and associated times of the detectors that fire. On the playback of the event data, observation of a pulse in the Ge(Li) detector in the photopeak of one of the $\gamma$-rays ensures that the second $\gamma$-ray was actually emitted inside the spectrometer in that event. By scanning the event data it is possible to construct the full single-photon and the $M$-photon response for the apparatus as described below.

5.1. Single-photon response methods

5.1.1. Spectrometer efficiencies

The single triggering efficiency for an incident photon of energy $E_0$, is defined as

$$\Omega = \sum_{i=1}^{N} \sum_{s} \Omega^{ij}(h_i),$$

(1a)

where $\Omega^{ij}(h_i)$ is the probability of recording a pulse height $h_i$ above a threshold $h_{th}$ in the $i$th detector, when no other detector responds. If all detectors are taken as equivalent then $\Omega = N \sum \Omega(h_i)$. Similarly, the double-triggering efficiency is

$$\Omega'' = \sum_{ij} \sum_{s} \Omega^{ij}(h_i, h_j),$$

(1b)

and the triple-triggering efficiency is

$$\Omega''' = \sum_{ijk} \sum_{s} \Omega^{ijk}(h_i, h_j, h_k) = N(N-1)(N-2) \sum_{s} \Omega(h_i, h_j, h_k).$$

(1c)

where $\Omega^{ij}(h_i, h_j)$ and $\Omega^{ijk}(h_i, h_j, h_k)$ are the probabilities for recording pulse heights $h_i, h_j, h_k$ above threshold in the $ij$ detectors, respectively; $\Omega(h_i, h_j)$ and $\Omega(h_i, h_j, h_k)$ are the probabilities for recording pulse heights $h_i, h_j, h_k$ averaged over all detector elements. The total triggering efficiency $\Omega_T$ is defined as

$$\Omega_T = \Omega + \Omega'' + \Omega''' + \cdots.$$

(2)

Clearly, $1 - \Omega_T$ is the probability for no response. We note that in eqs. (1a–c) each multiple event must be counted only once.

Calibration measurements are normally taken with the following $\gamma$-ray sources: $^{75}$Se (136–265 keV cascade), $^{207}$Bi (1064–570 keV cascade), $^{60}$Co (1173–1332 keV cascade), $^{88}$Y (898–1836 keV cascade), and $^{24}$Na (2754–1369 keV cascade). The results for $\Omega_T$, $\Omega'$, $\Omega''$ and $\Omega'''$, corrected for random events, branching ratios and internal conversion are shown in fig. 7 for one such calibration with 69 detectors in the spectrometer [23]. It is seen that $\Omega''$ rises rapidly above $\sim 300$ keV, levels off and reaches the value of 0.30 at 2754 keV, whereas $\Omega'''$ is small up to $\sim 2$ MeV ($<0.83$), but above this energy it increases rapidly.

Fig. 7. Measured detection efficiencies for a setup with 69 detectors in the spectrometer. The open squares, closed squares, open triangles and closed triangles give the triggering efficiencies for 1-, 2-, 3- and 4-fold events, respectively, from monoenergetic $\gamma$-rays as a function of $\gamma$-ray energy. The open circles give the total triggering efficiency. The closed circles give the fraction of $\gamma$-ray energy deposited in the entire spectrometer.
5.1.2. Scattering factors

Following ref. 15 we define the double and triple scattering factors \( F'' \) and \( F''' \) by \( F'' = \Omega''/\Omega_T \) and \( F''' = \Omega''''/\Omega_T \), so that

\[
\Omega' = \Omega_T (1 - F'' - F''' - \cdots) = \Omega_T (1 - F),
\]
where
\[
F = F'' + F''' + \cdots.
\]

The angular dependence of the two-fold scattering is given by

\[
F''(\theta) = \sum \Omega''_{ij}/\Omega_T, \tag{5}
\]
where \( \Omega''_{ij} \) is defined as \( \Omega''_{ij} = \sum h_{ij} h_{k} \) and the summation is over a subgroup of detectors having an angle \( \theta \) with the \( i \)-th one. If the \( j \)-th detector is taken to be one of the pentagonal elements in the spectrometer, then there are 14 groups of 5 detectors forming the following angles with it: 24.4°, 41.6°, 49.5°, 63.4°, 67.9°, 77.5°, 87.3°, 92.7°, 102.5°, 112.1°, 116.6°, 130.5°, 138.4°, and 155.6°. In this case eq. (5) defines 14 values of \( F''(\theta) \). Earlier measurements of \( F''(\theta) \) for the 662 and 854 keV \( \gamma \)-rays reported in ref. 24 agree well with the present results. It is worth pointing out that for the 854 keV \( \gamma \)-rays about 72\% of the detector-to-detector scattering is concentrated among the five nearest neighbors [24]. For \( \theta \geq 60° \) the scattering is very small and essentially independent of angle.

The scattering factors \( F, F'', F''' \), etc. defined in eq. (4) permit evaluation of the efficiencies \( \Omega', \Omega'', \Omega''' \), etc. from the total triggering efficiency \( \Omega_T \) via eqs. (3) and (4). Another important quantity is the fold scattering factor \( F_k \), which is defined as

\[
F_k = F'' + 2 F''' + 3 F'''' + \cdots = (\Omega'' + 2 \Omega''' + 3 \Omega'''' + \cdots)/\Omega_T. \tag{6}
\]

Knowledge of \( F_k \) permits one to calculate the number of detectors that fire for an input of \( M \) \( \gamma \)-rays when the total triggering efficiency is \( \Omega_T \). The quantity \( F_k \) used in ref. 15 becomes the same as \( F_k \) when higher order terms are added to it.

With the above definition of \( F_k \), eq. (25) of ref. 15 gives

\[
\langle k \rangle = N \left[ 1 - \left( \frac{\Omega_T (1 + F_k)}{N} \right)^M \right]. \tag{7}
\]

The corrected total triggering efficiency \( \Omega_T (1 + F_k) \) can be expressed in terms of \( \Omega', \Omega'', \cdots \) by using eqs. (2) and (6):

\[
\Omega_T (1 + F_k) = \Omega' + 2 \Omega'' + 3 \Omega''' + \cdots. \tag{8}
\]

Expression (7) is applicable only to a cascade of \( M \) equal-energy \( \gamma \)-rays. For a cascade of \( M \) \( \gamma \)-rays with energies \( \langle E_k \rangle = 1, \ldots, M \) it should be written as

\[
\langle k \rangle = N \left[ 1 - \prod_{k = 1}^{M} \left( 1 - \frac{\Omega_T (1 + F_k)}{N} \right) \right]. \tag{9}
\]

where the efficiencies in parentheses are evaluated at the appropriate transition energies \( E_k \). It should be noted that \( \Omega_T (1 + F_k) \) can be larger than unity for sufficiently large values of \( F_k \). Using the values from the smooth curves drawn through the data of fig. 7 we obtain the results for \( \Omega_T (1 + F_k) \) shown in fig. 8. The rapid increase observed over the energy range from 300-800 keV is associated with the sharp rise of \( \Omega'' \) observed over the same energy range (see fig. 7). From 1.0 to 2.2 MeV \( \Omega_T (1 + F_k) \) is essentially independent of energy, but above 2.2 MeV it increases again as a result of the onset of triple scattering which is primarily due to pair production. The large size of the individual detectors in this spectrometer suppresses considerably the double escape peaks in the spectra of the individual detectors and for this reason \( \Omega'''' \) does not rise significantly until \( E_k \geq 2.5 \) MeV. In most heavy-ion fusion reactions practically all of the entry-state population occurs for values of \( E/M \) within the flat portion of \( \Omega_T (1 + E_k) \) in fig. 8. Thus, approximate but realistic calculations can be made with \( \Omega_T (1 + F_k) = 1.22 \) which corresponds to an \( E_k \) of 1.2 MeV, for which \( \Omega_T = 0.91 \) and \( F_k = 0.34 \). It should be noted that the experimental values of \( F_k = 0.30 \) for \( E_k = 1.0 \) MeV and \( F_k = 0.66 \) for \( E_k = 2.75 \) MeV are considerably higher than the estimate of 0.25 for \( F_k \) used in the calculations of ref. 15.

The variation of \( \langle k \rangle \) with \( E_k \) for \( M = 10, 20, 30 \) is shown in fig. 9, while the dependence of \( \langle k \rangle \) on \( E/ME_k \) for constant \( M \) is illustrated in fig. 10. The general shape of \( \Omega_T (1 + F_k) \) (see fig. 8) is reflected in both these figures. The variation of \( \langle k \rangle \) with \( M \) for the same value 1.22 of \( \Omega_T (1 + F_k) \) is shown in fig. 11. This result is in good agreement with the predicted behavior shown in fig. 8 of ref. 15 which was calculated for \( \Omega_T (1 + F_k) = 1.19 \).

The validity of the equal-energy approximation will
be examined next. We consider as an example a cascade consisting of the yrast band in $^{160}$Yb up to $\ell = 28$ [25] and have extended it to $J = 60$ assuming a rotational spectrum given by $E_J = (4J - 2)/132$ MeV. A total energy of $E = 30.0$ MeV at $M = 30$ is thus obtained. Using eq. (9) we find $\langle k \rangle = 27.6$. The equal-energy approximation with $\Omega_E(1 + F_k)$ evaluated at the average energy $E/M = 1.0$ MeV gives, according to eq. (7), $\langle k \rangle = 28.6$. This 3.6% difference is due to the presence of 13 transitions in the region below 800 keV, where $\Omega_E(1 + F_k)$ increases rapidly with energy.

5.1.3. Generalized efficiencies

The spectral functions defined by eq. (40b) of ref. 15 characterize completely the moments of the $P_H(E,M,h_k)$ response with respect to the total pulse height $H$. Here we report experimental measurements of the first three spectral functions $\omega_l(E)$ (for $l = 1, 2$ and 3) which are defined by

$$\omega_l(E) = \sum_{H_k} \Omega_H(E, H_k) H_k^l$$

(10)

where $\Omega_H(E, H_k)$ is the efficiency of the entire spectrometer to record a total pulse height $H_k$ from an incident γ-ray of energy $E$. In fig. 12b we show spectra recorded with the entire spectrometer for the indicated incident γ-ray energies. These spectra give the efficiencies $\Omega_k(E, h_k)$ which in turn characterize completely the spectral functions in eq. (10). For comparison, fig 12a shows the pulse-height spectra $\Omega_k(E, h_k)$ from the individual NaI detectors in the spectrometer, for the same incident γ-ray energies as indicated. The efficiency $\Omega_k(E, h_k)$ is the sum of the pulse-height spectra $\Omega_k^{(i)}(E, h_k)$ of all the NaI detectors ($i$) from 1 to $N$. The spectra for fig 12a differ from those defined by $\Omega_k^{(i)}(h_k)$ in eq. (1) because they include the scattering in from the other detectors in the spectrometer. Clearly

$$\Omega_k(1 + F_k) = \sum_k \Omega_k(E, h_k)$$

$$= \sum_{i=1}^N \sum_k \Omega_k^{(i)}(E, h_k).$$

(11)

Fig. 11. Measured average coincidence fold $\langle k \rangle$ as a function of $M$ for 1.2 MeV γ-rays.

Fig. 12. (a) Pulse-height spectra recorded with individual NaI detectors in the spectrometer for each of the incident γ-ray energies indicated. These give the efficiencies $\Sigma \Omega_k^{(i)}(E, h_k)$ in thousands of counts/10 keV for 50000 incident photons. (b) Total pulse-height spectra recorded with the entire spectrometer for the indicated incident energies. These give $\Omega_k(E, h_k)$ in thousands of counts/10 keV for 50000 incident photons as in (a).

Fig. 9. Measured average coincidence fold $\langle k \rangle$ for $M = 10, 20$ and 30 equal-energy γ-rays, as a function of γ-ray energy.

Fig. 10. Measured average coincidence fold $\langle k \rangle$ as a function of total γ-ray energy $E$ for the indicated values of multiplicity $M$. 
A set of generalized efficiencies $\Omega_{E_\gamma}^{(l)}$ may be defined by

$$\Omega_{E_\gamma}^{(l)} = \omega_l(E_\gamma)/E_\gamma^l,$$

(12)

where, for example, $\Omega^{(0)}_T$ is the triggering efficiency $\Omega_T$ and $\Omega^{(1)}_F$ gives the fraction of the energy recorded by the entire spectrometer for an incident $\gamma$-ray of energy $E_\gamma$. We note that the efficiencies $\Omega^{(l)}_F$ decrease with increasing $l$ for constant $E_\gamma$ [15].

The generalized efficiencies $\Omega^{(l)}_F$ for $l = 1, 2$, and 3 obtained from the data of fig. 12b are shown in fig. 13. From these efficiencies the spectral functions $\omega_l(E_\gamma)$ can be computed via eq. (12). In turn, the latter can be used to compute the first three moments of the projection of the response $P_{\omega l}(EM, HK)$ on the $H$ axis [15]. The first two moments in $H$ are given by the following expressions, where the second equalities apply to an equal-energy cascade with $E_\gamma = \sum E_{\gamma_k}/M$:

$$\langle H \rangle = \sum_{k=1}^M \omega_k(E_\gamma) = M \omega_1(E_\gamma),$$

(13)

$$\sigma_H^2 = \sum_{k=1}^M \left[ \omega_2(E_\gamma) - \omega_1(E_\gamma)^2 \right]$$

$$= M \left[ \omega_2(E_\gamma) - \omega_1(E_\gamma)^2 \right].$$

(14)

In terms of the generalized efficiencies the first two moments $\langle H \rangle$ and $\sigma_H^2$ are given by

$$\langle H \rangle = \sum_{k=1}^M \Omega_{E_\gamma}^{(1)} E_\gamma = E \langle \Omega_{E_\gamma} \rangle,$$

(15)

$$\sigma_H^2 = \sum_{k=1}^M \left( \Omega_{E_\gamma}^{(2)} - \Omega_{E_\gamma}^{(12)} \right) E_\gamma^2,$$

(16)

where $\langle \Omega_{E_\gamma} \rangle = \langle H \rangle / E$, is the average efficiency. It is useful to approximate this quantity by the efficiency $\Omega_{E_\gamma}$ at the average energy $E_\gamma = E/M$ for a cascade of $M$ equal-energy $\gamma$-rays. That is

$$\langle H \rangle = E \langle \Omega_{E_\gamma} \rangle = M E_\gamma \Omega_{E_\gamma}.$$  

(17)

This approximation works better for the evaluation of $\langle H \rangle$ for a realistic cascade, than for the evaluation of $\langle k \rangle$ for the same cascade. This is because $\Omega_{E_\gamma}^{(1)}$ varies less with $E_\gamma$ (essentially linear) than $\Omega_T(1 + E_\gamma)$ (compare figs. 13 and 8). For the $^{166}$Yb example given in the previous section, we find $\langle H \rangle = 23.9$ MeV which gives $\langle \Omega_{E_\gamma} \rangle = 0.797$. The equal-energy approximation gives $\langle H \rangle = 24.3$ MeV or $\langle \Omega_{E_\gamma} \rangle = 0.810$, a difference of only 1.7%. In view of this result, eq. (17) with $\Omega_{E_\gamma} = \Omega_{E_\gamma}^{(1)}$ was used to obtain the dependence of $\langle H \rangle$ on $E$ for constant values of $M$, and the dependence of $\langle H \rangle$ on $E$ for constant values of $E$ shown in figs. 14a and b. It is clear from fig. 14a that $\langle H \rangle$ is insensitive to $M$ for constant total energy $E$. Similarly, the variation of $\langle H \rangle$ with $E$ shown in fig. 14b exhibits only a small dependence on $M$.

5.1.4. The mappings $(E, M) \rightarrow (\langle H \rangle, \langle k \rangle)$ and $(H, k) \rightarrow (\langle E \rangle, \langle M \rangle)$

For a given input $\gamma$-ray spectrum $(E_\gamma)_{1 \ldots M}$ with $E_\gamma = \sum_{i=1}^M E_i$ the mapping $(E, M) \rightarrow (\langle H \rangle, \langle k \rangle)$ is well defined from eqs. (13) and (9) in terms of $\Omega_{E_\gamma}$ and

$$\Omega_T = \Omega_{E_\gamma}^{(0)}$$

for each $\gamma$-ray energy. In contrast, the reverse mapping $(H, k) \rightarrow (\langle E \rangle, \langle M \rangle)$ cannot be derived in general because the average cascade path associated with a given $(H, k)$ selection is not known. However, with the equal energy approximation it is possible by the following iterative procedure to find a rapid solution incorporating the energy dependence of $\Omega_T(1 + F_\gamma)$ (see fig. 8) and of $\Omega_{E_\gamma}^{(1)}$ (fig. 13). Assume for the first trial that $M_1 = k$, $E_1 = H/0.8$ and $E_{\gamma_1} = E_\gamma/M_1$. From figs. 8 and 13 we obtain $[\Omega_T(1 + F_\gamma)]_2$ and $\Omega_{E_\gamma}^{(1)}$. Using these results and solving eq. (7) for $M$ we find

$$M_2 = \frac{\ln(1 - k/N)}{\ln\left[ 1 - \left[ \Omega_T(1 + F_\gamma) \right]_1 / N \right]}$$

(18)
and $E_s = H_1 / \Omega_{(E_s)}$. From these results we find $\langle E_s \rangle_2 = E_s / M_s$. Now we repeat the procedure, until successive values of $(E, M)$ do not differ significantly. For entry-state populations resulting from typical heavy-ion fusion reactions [23] we have observed that $H/k$ varies between 0.3 and 1.8 MeV. In such cases this procedure converges rapidly, typically after two iterations.

The reverse mapping $(H, k) \rightarrow (\langle E \rangle, \langle M \rangle)$ can also be obtained from a complete set of reverse responses (section 5.3).

5.2. Multiple-photon responses $P_N(EM, Hk)$

In order to construct the response functions $P_N(EM, Hk)$ we start with the event data from a source with two $\gamma$-rays. Data are recorded with the spectrometer triggered by a Ge detector. The event tapes are scanned with gates on the full-energy peaks in the Ge spectrum and on the prompt peak of each Ge–NaI time spectrum for the coincident events. The events in the full energy gate ensure that the other $\gamma$-ray was emitted inside the spectrometer. The events satisfying these criteria are copied and used to create new simulated events of multiplicity $M$. To construct the response for, say, $M = 30$ and $E = 30 \cdot E_s$ ($E_s$ is the energy of the $\gamma$-ray that goes to the spectrometer) we scan the new events in groups of 30. Each group has all the characteristics of a 30-photon event, namely, some $\gamma$-rays may escape the spectrometer, some may hit the same detector more than once causing a coincident sum which is counted only once, and others may trigger several detectors. The total pulse height $H$ is obtained by summing the pulse heights of all the NaI elements that fired, while the fold $k$ is given by the number of detectors that fire in each group of 30. The events with pulse heights in the Ge gate, which do not have any NaI triggers within the time gates measure the number of $\gamma$-rays that escape detection. Typically, at least 10,000 or more events of each multiplicity must be created. Responses for $(E, M)$ with $E = M \cdot E_s$ have been constructed for $E_s = 136, 265, 570, 898, 1173, 1332, 1369, 1836$ and 2754 keV using the sources listed in section 5.1.1. Responses for intermediate $E$ values can be obtained by linear interpolation from the measured responses $P_N(E_i, M, Hk)$ and $P_N(E_2, M, Hk)$, where $E_i < E < E_2$.

First, the centroid of $P_N(EM, Hk)$ is calculated by interpolation from the centroids $(\langle H_1 \rangle, \langle k_1 \rangle)$ and $(\langle H_2 \rangle, \langle k_2 \rangle)$, then the centroids of the calibration responses are shifted to $\langle H \rangle, \langle k \rangle$ and the counts are interpolated point by point. Figs. 15 and 16 show density plots of the responses $P_{\gamma\gamma}(EM, Hk)$ with $M = E / (1.0 \text{ MeV})$ for $E = 10, 25, 40$, and $5, 20, 35$ and 50 MeV, respectively. Differences between the values of $\langle k \rangle$ from the responses of fig. 16 and those calculated using the $\Omega_{\gamma} (1 + F_k)$ curve of fig. 7 in eq. (7) are smaller than 0.5 unit, which is well within the uncertainties in the measured efficiencies. The behavior of these responses is further illustrated by their projections on the $k$ and $H$ axes. Fig. 17 shows the projections on the $k$
The first two moments of the distributions in \( H \) and \( k \), shown in figs. 17 and 18, are in good agreement with the values calculated using the results of the single-photon responses. The fractional resolutions \( \Delta k/\langle k \rangle \) and \( \Delta H/\langle H \rangle \) (where \( \Delta k \) and \( \Delta H \) are the fwhm values) from the multiple-photon responses of figs. 17 and 18 (data points) and those calculated using eqs. (22)–(24) and (32) or ref. 15 (smooth curves) are compared in figs. 19a and 19b, respectively. The smooth curves were calculated with the efficiency results shown in figs. 7, 8 and 13. The two methods agree well.

It is clear that construction of multiplicity-\( M \) events by grouping \( M \) consecutive single events give poorer statistics for high \( M \). However, the total number of counts, \( N \), recorded during calibration is usually very large (for a typical data set \( N > 10^5 \)). Since \( N \) single events can be grouped into \( M \) events in \( N!/(N-M)! \) ways, it is possible to use random sampling from the \( N \) events to obtain an equal number of events for every choice of \( M \) with only a very small fraction of the possible combinations being sampled. The spectral characteristics in each NaI element are well sampled with a total of \( N > 2 \times 10^5 \) counts, and this number should give an adequate description of the responses.

The present method for constructing the \( M \)-photon response can be extended to any given input spectrum \( \langle E_i \rangle_{i=1,\ldots,M} \). One can approximate rather well the input spectrum by a suitably weighted combination of events from the different \( \gamma \)-ray sources, and then construct the bivariate response \( P_h(EM, HK) \) from the combined events. The effect of the \( E_x = E/M \) approximation on the \( \langle k \rangle \) and \( \langle H \rangle \) values was discussed in sections 5.1.2 and 5.1.3. Here we should point out the effect on the widths \( \Delta k \) and \( \Delta H \). For the example of \( ^{169}\text{Yb} \) in section 5.1.2 \( \Delta k_{\text{opt}}/\Delta k_{E/M} = 0.974 \), while \( \Delta H_{\text{opt}}/\Delta H_{E/M} = 1.146 \). That is, the equal-energy approximation significantly underestimates the width in \( H \). Furthermore, for a given \( (E,M) \) point the width of \( P_h(EM, HK) \) in \( H \) for constant \( k \) is about 30% smaller with this approximation than for the example of a rotational spectrum.

5.3. Reverse responses – Unfolding of \( (H, k) \) distributions

It is often necessary to determine the \( (E, M) \) distributions in total energy \( E \) and multiplicity \( M \) associated with the selection of events between \( H \) and \( H + \delta H \) having a given coincidence fold \( k \). For this purpose we create the reverse response which is the distribution in the \( (E, M) \) space that contributes to a given point \( (H, k) \). Assuming for the moment that all \( (E, M) \) points are equally probable, we can construct the reverse responses \( P_h(Hk, EM) \) from a complete set of responses \( P_h(Hk, EM) \) by regrouping and normalizing. Normalization is important because for a given \( E \), \( dM/dk \) is a
function of \( k \) or of \( M \) (see eq. (7)):

\[
\frac{dk}{dM} = -N \left[ 1 - k \right] \ln \left[ 1 - \frac{\Omega_T(1 + F_k)}{N} \right].
\]

(19)

whereas \( dE/dH \) is equal to \( \langle \Omega_E \rangle \) and thus depends only on \( E \), and is not a function of \( H \).

The dependence of the reverse responses \( P_N(H, E, M) \) on \( E \) and \( M \) is shown in figs. 20 and 21, which illustrate the projections on the \( M \) and \( E \) axis, respectively, for several values of \( k \) and \( H \) as indicated. In the examples in figs. 20 and 21 we chose the \( H \) and \( k \) values such that \( H = k \) in MeV.

The reverse responses \( P_N(H, E, M) \) can be used to specify completely a region in the \( (E, M) \) space corresponding to any \( (H, k) \) choice, provided the population distribution \( R_x(E, M) \) for a class of events \( x \) is known. The selected region \( \Delta R_x(E, M) \) is then obtained as the product

\[
\Delta R_x(E, M) = R_x(E, M) P_N(H, E, M).
\]

(20)

Here we must point out that the reverse responses \( P_N(H, E, M) \) are not the inverse of \( P_N(E, M, H) \). The populations \( R_x(E, M) \) must be obtained from the measured populations \( Q_x(H, E, M) \) by solving the system of equations:

\[
Q_x(H, E, M) = \sum_{E, M} P_N(E, M, H) R_x(E, M).
\]

(21)

We have solved this system of equations by an iterative least-squares unfolding procedure. As an initial estimate for \( R_x(E, M) \) we use the input data shifted by the reverse mapping \( (H, k) \rightarrow (\langle H \rangle, \langle M \rangle) \), to the \( (E, M) \) space. Since in this mapping the \( \langle (E), \langle M) \rangle \) values are not located exactly at the grid points of \( R_x(E, M) \), we distribute the counts from each \( (H, k) \) point to a small region around the corresponding point, \( \langle (E), \langle M) \rangle \), using a narrow bivariate Gaussian function. The width of this Gaussian distribution must be kept significantly smaller than the responses \( P_N(E, M, H) \) in order to avoid broadening in the estimate \( R_x(E, M) \). The first fit \( F_1(H, k) \) to the data is obtained using eq (21) as

\[
F_1(H, k) = \sum_{E, M} P_N(E, M, H) R_x(E, M).
\]

(22)

The fit \( F_1(H, k) \) is compared point by point with the data and a ratio matrix is calculated as \( B_1(H, k) = R_x(E, M)/F_1(H, k) \), where the values \( R_x(E, M) \) are connected to \( B_1(H, k) \) via the reverse mapping. The first estimate is then multiplied point by point by each of the reverse responses \( P_N(H, E, M) \) to give

\[
P_N(H, E, M) = \frac{R_x(E, M) P_N(H, E, M)}{\sum_{E, M} R_x(E, M)}.
\]

(23)

The centroids of these new reverse responses, \( P_N(H, E, M) \), provide now a better estimate for the reverse mapping. The second estimate \( R_2(E, M) \) is obtained next by multiplying the data \( Q_x(H, k) \) by \( B_1(H, k) \) according to the mapping \( (H, k) \rightarrow (\langle E \rangle, \langle M \rangle) \) from the reverse responses \( P_N(H, E, M) \). If desired, a second improvement in the reverse mapping may be derived by calculating new reverse responses as \( P_N''(H, E, M) = R_2(E, M) P_N'(H, E, M) / \sum_{E, M} R_2(E, M) \). The procedure is repeated until a minimum in the chi-square for the fit to the data is obtained. Usually this is achieved after two iterations with adjusted reverse mapping and one or two additional iterations with fixed mapping.

As an example of the results from the unfolding procedure we show in figs. 22a and b the data from the entry state population \( Q_{\text{tot}}(H, k) \), and the unfolded population \( R_{\text{tot}}(E, M) \) from the reaction 136 MeV \(^{20}\text{Ne} + ^{146}\text{Nd} = ^{160}\text{Yd} + \text{6n} \).

As expected the distribution \( R_x(E, M) \) falls off rapidly at the tails of the \( (E, M) \) distribution. Consequently, the distribution \( \Delta R_x(E, M) \) corresponding to a gate \( (H, k) \) is shifted considerably toward the most probable \( (E, M) \) value compared to \( P_N'(H, E, M) \).
5.4. Spectral response of a single NaI detector

The energy spectrum of $\gamma$-rays for a given class of events, e.g. those associated with a given $(H, k)$ selection, can be studied by placing a separate NaI detector outside the spectrometer to record the pulse-height spectrum coincident with the spectrometer and the triggering device(s). The large distance of the separate detector combined with appropriate shielding ensures absence of coincidence summing or crystal-to-crystal scattering. In this way the associated energy spectrum can be derived by unfolding the pulse-height spectrum using appropriate single-photon responses for the outside detector. At the maximum resolving power of the spectrometer, i.e. events between $H$ and $H + \delta H$ with $\delta H = 1.0$ MeV and a single value of $k$, such measurements always suffer from poor statistics. A spectrum recorded with one detector in the spectrometer offers $\sim 11$ times better statistics compared to, say, a 12.7-cm diameter by 15.2-cm long NaI detector at 90 cm from the target. By adding the spectra from all 70 elements, one obtains $\sim 760$ times more counts. This makes it highly desirable to measure $\gamma$-ray spectra with the detectors in the spectrometer. In order to do this, however, one must apply corrections for coincidence summing and for detector-to-detector scattering because these effects distort the observed spectra considerably. These corrections can be applied to the observed pulse-height spectra or they can be incorporated into the single-photon responses for each detector. The latter approach has been adopted in this work. The effect of scattering is easily determined quantitatively, by noting that the procedure described in section 5 for obtaining the single-photon response parameters also provides the spectra that include the scattering from the calibration sources listed in section 5.1.1.

The effect of coincidence summing is a function of the $\gamma$-ray multiplicity, $M$, and of the shape of the observed spectrum itself. The amount of summing for a given $M$ and a total $\gamma$-ray energy $E$ can be measured by the same method for grouping single-photon events that was used to obtain the multiple-photon responses $P_n(EM, HK)$ in section 5.2. The simulated events of multiplicity $M$ can be scanned to provide the average summing as a function of $M$. For a given event of multiplicity $M$, we observe $k_b$ pulses in $k$ detectors that fired, where $k_b > k$. The differences $s = (k_b - k)$ from $N$ events of multiplicity $M$ show a distribution $I(s)$. The average summing fraction is thus defined by

$$S(E, M) = \left[ \frac{\sum_s s I(s)}{M \cdot \sum_s I(s)} \right]$$

where $E = E_y \cdot M$.

The average summing defined this way can be constructed for each calibration source. From these values a complete map for every $(E, M)$ point can be obtained by interpolation. The average summing as a function of $M$ is shown in fig. 23 for $E_y$ values of 570, 898, 1836, and 2754 keV. It is seen that in addition to the strong dependence on $M$, there is a dependence on $\gamma$-ray energy similar to that of $\Omega_\gamma(1 + F_\gamma)$ (see fig. 8).

The single-photon spectra that include the scattering can be modified to include also the summing. Clearly, the summing alters the shape of the spectrum by shift-
6. Performance of the spectrometer

The response of the entire spectrometer to γ-rays and to neutrons from heavy-ion induced reactions has been investigated in order to understand the performance of the instrument and to optimize the technique for identification of the neutrons from their time of flight.

6.1. Identification of γ-rays and neutrons

The large cross section for inelastic scattering of neutrons on iodine and the large thickness of the NaI detectors leads us to expect large efficiency for detection of neutrons. The timing resolution of the detectors in the spectrometer (fig. 5b) is sufficient to allow identification of the neutrons by time of flight. The energy dependence of the neutron TOF for target to interaction-point distances of 18, 27 and 38 cm is shown in fig. 12 of ref. 24. From that figure it was deduced that pulses from neutrons with energies less than 7, 17 and 30 MeV interacting in the front, the middle or the back of a NaI element can be distinguished from γ-ray pulses.

The spectrometer has been used in experiments with the ORIC and with the 25 MV tandem accelerator at the Holified Heavy-Ion Research Facility. The time resolution of the beam microstructure at the cyclotron is typically ~3.5 ns, while the Tandem at present provides only a continuous beam. In order to make full use of the timing resolution of each NaI detector, we must either use triggering detectors with timing resolution significantly better than 2 ns or establish the time of each event from the distribution of the triggering times in the spectrometer itself. We have developed a technique based on the latter method that works satisfactorily for events with γ-ray coincidence fold k ~ 3.

In order to define the zero time, t0 of a nuclear event an iterative search is initiated to identify a cluster of prompt γ-pulses. Initially, the times from all the NaI detectors are shifted by previously determined amounts so as to match the detectors to ± 0.2 ns. Then the times from the detectors that fired within a long interval, typically 25–50 ns, centered at the most probable time, are averaged in order to obtain the first estimate t2 for t0. Clearly, if some neutrons were recorded within this interval then t2 would be too high. In the second iteration the sampling interval is reduced asymmetrically to be from t2 − 10 to t2 + 5 ns, and the times of the detectors firing within this interval are averaged to give t1. Since a good part of the neutron pulses that arrive later have been left out, t1 is a better estimate of t0. In the third and last iteration the sampling interval is further reduced to be from t1 − 4 to t1 + 4 ns, and the times of the detectors within this interval are averaged to give t0. For convenience, the times of all the detectors are shifted so that the t0 for that event is at some selected time t1, which is taken to be the same for all the
events. From these corrected time spectra, it is possible to decide with improved timing resolution whether each pulse was due to a neutron or a γ-ray by reference to a selected mask. The above procedure defines the \( t_0 \) of an event with an accuracy of \( \Delta t/\sqrt{k} \), where \( \Delta t \) is the average time resolution of the \( k \) detectors that fired within the last sampling gate.

In order to demonstrate the adequacy of the \( t_0 \) technique we show in fig. 24a a density map of a pulse height vs. time spectrum taken in one of the detectors at 24° from an experiment with 149 MeV \(^{20}\)Ne on \(^{146}\)Nd. The starting time is derived from a Ge(Li) detector. The limited timing resolution does not permit a clear separation of the neutron events from the γ-rays. Fig. 24b shows a map of the same data with the times measured with respect to \( t_0 \) obtained as described above. The improvement is apparent. The time spectra selected through the rectangular masks indicated in figs. 24a and b are shown in fig. 25 and correspond to a pulse-height interval 360–520 keV. The upper fig. 25b is the spectrum from the corrected map.

In order to demonstrate further how the \( t_0 \) technique works as a function of \( k \), we show in figs. 26a–e time spectra corresponding to pulse heights between 580–700 keV created from maps with γ-coincidence fold \( k_\gamma \) of 5, 10, 16, 22, and 28, respectively. These events were triggered with any count in the Ge(Li) detector without any exit channel selection. It is seen that as the fold \( k_\gamma \) increases the relative number of neutron pulses between 580–700 keV increases at first and then decreases for \( k_\gamma > 10 \). For this pulse-height gate and the above listed values of \( k_\gamma \), the fraction of neutron pulses was found to be 31, 38, 28, 20, and 14%, respectively. At larger pulse heights the fraction of neutron pulses increases considerably. In figs. 26f–j we show time spectra corresponding to pulse heights between 860–1220 keV obtained from the same maps with \( k_\gamma \) of 5, 10, 16, 22, and 28, respectively. The time resolution improves with in-
creasing pulse height as indicated. For this pulse-height interval the fraction of neutrons was found to be of 29, 53, 51, 37, and 31%, respectively.

The pulse-height spectra of the neutrons exhibit considerable line structure due to the inelastic excitation of $^{127}$I and $^{23}$Na in the detectors. A particularly large number of late pulses with large amplitude is observed in the $h$ vs. $t$ maps (see fig. 24b). The pulse-height spectra from the late pulses show a typical structure of neutron capture $\gamma$-rays with many low lying lines and several ones near the neutron binding energy.

6.2. Correlation between the neutron and $\gamma$-ray response

The high efficiency of the spectrometer to record neutron events suggests that the distribution of the population of total neutron pulse heights $H_n$ should be strongly correlated with the neutron coincidence fold $k_n$, in a manner similar to the total pulse height $H_n$ for $\gamma$-rays and their fold $k_\gamma$. The response of the spectrometer to neutrons was discussed in some detail in ref. 15. The average neutron fold $k_n$ was shown [15] to be given by

$$\langle k_n \rangle = N \left( 1 - \bar{\Omega}_n (1 + F_n) \right)^x$$

(25)

where $\bar{\Omega}_n$ is the average detection efficiency for neutrons per detector, $F_n$ is the neutron scattering factor and $x$ is the neutron multiplicity. Expressions for the second and third moment for the distribution in $k_n$, as well as for the distribution in total neutron pulse height $H_n$ and its second and third moments are given in ref. 15.

We have used the reaction 136 MeV $^{20}$Ne on $^{146}$Nd and triggered the spectrometer with a Ge detector. In fig. 27a we show as a shaded contour map the population $Q_n(H_n, k_n)$. The different contours correspond to decreasing factors of two going outward from the center.
Fig. 27. Density and contour maps from the reaction of 136 MeV $^{20}$Ne on $^{146}$Nd (ref. 23) before unfolding. The spectrometer was triggered by any pulse in the Ge detector. Each contour line signifies a decrease in intensity by a factor of two going outward. (a) $\gamma$-ray population $Q_{\gamma}(H_{\gamma}, k_{\gamma})$. (b) Neutron distribution $Q_{n}(H_{n}, k_{n})$. (c) Population distribution for $Q_{\gamma+n}(H_{\gamma+n}, k_{\gamma+n})$. It is considerably wider in both dimensions. (d) Correlation of $k_{n}$ with $k_{\gamma}$. (e) Correlation of $H_{\gamma}$ with $H_{n}$.

In Fig. 27b we show a similar map for $Q_{n}(H_{n}, k_{n})$. It is seen that at 1/10 maximum as many as 13 detectors respond to neutrons when the average neutron multiplicity from this reaction is $\approx 6.2$. In fig. 27c we show the event-by-event correlated distribution for both neutrons and $\gamma$-rays $Q_{\gamma+n}(H_{\gamma+n}, k_{\gamma+n})$. The composite distribution is considerably wider along the $k_{\gamma+n}$ and $H_{\gamma+n}$ axes. The correlation of the $\gamma$-ray and neutron coincidence folds $k_{n}$ and $k_{\gamma}$ is shown in Fig. 27d. It is seen that the most probable value for $k_{n}$ corresponds to a $k_{\gamma}$ of 20. In Fig. 27e we show the correlation of $H_{\gamma}$ with $H_{n}$. For $H_{\gamma} \geq 14$ MeV the observed $H_{n}$ values decrease with increasing $H_{\gamma}$.

The projections of the fold axis of the populations from Figs. 27a–c are shown in Fig. 28a. The closed squares, open circles and closed circles show the distributions for $k_{\gamma}$, $k_{n}$, and $k_{\gamma+n}$, respectively. It is clearly seen that the combined distribution for $k_{\gamma+n}$ is shifted upward from $k_{\gamma}$ by about 6 units, which is close to the value for $\langle k_{\gamma} \rangle$. Using the projected distribution for $k_{\gamma}$, and the known value of 6.2 for the neutron multiplicity, we find from eq. (25) that $\hat{d}_{\gamma}(1 + F_{\gamma}) = 0.0153$, which gives the value of 1.06 for the corrected total neutron-triggering efficiency $\hat{d}_{\gamma}(1 + F_{\gamma})$. This is comparable with the

value of 1.2 for $\Omega_{\gamma}(1 + F_{\gamma})$ for $\gamma$-rays obtained from the flat region in fig. 8.

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References

[18] LeCroy Research Corp. Model 612A, 12-channel linear amplifiers.
[22] LeCroy Research Corp. No. HV4032A 32-Channel High Voltage Supply and Model 2132 CAMAC Interface.