Spin and Temperature Dependence of Nuclear Deformation
Using Alpha-Gamma Angular Correlations

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ABSTRACT

Alpha-particle angular distributions with respect to the spin direction of residual nuclei have been measured in heavy-ion fusion reactions. The spin direction was determined by measuring the $\gamma$-ray angular distributions, for each event, using the spin spectrometer. $\alpha$-particle anisotropies have been extracted for the compound nuclear systems: $^{110}$Sn* (94 MeV), $^{114}$Sn* (80 MeV), $^{138}$Nd* (82 MeV), $^{164}$Yb* (67 MeV) and $^{170}$Yb* (135 MeV) as a function of the $\alpha$-particle energy and $\gamma$-ray multiplicity. The results are compared with statistical model calculations using transmission coefficients from a spherically symmetric optical model potential. The trend of the anisotropy coefficients below the evaporation Coulomb barrier is consistent with spherical emitting shapes in the case of the Sn* isotopes. Small deformation effects are suggested by the $^{138}$Nd* and $^{164}$Yb* data. The $^{170}$Yb* data indicate a large deformation which increases considerably with increasing spin. These results are in agreement with findings for similar systems in which the decay of the giant resonances built on excited states has been studied.
1. INTRODUCTION.

The study of nuclear shapes at high angular momentum and excitation energy is a topic of current extensive theoretical and experimental interest in heavy-ion physics. A number of giant resonance studies have tried to explore the effect of high excitation and/or angular momentum degrees of freedom on the nuclear shapes. However, the fact that (fission-stable) compound nuclei with the highest possible angular momentum often decay by emitting alpha particles has suggested the study of $\alpha$-particle emission as an alternate probe of nuclear shapes. If the highly excited and rapidly rotating compound nucleus is deformed, it exhibits an evaporation barrier for charged-particle emission lower than that of the spherical case. Consideration of deformation makes substantial changes in the transmission coefficients $T_1$ for particle emission. Since the residual excitation has an exponential influence on the level densities, this change in $T_1$ values leads to a strong enhancement of $\alpha$-decay, especially in the energy region below the evaporation Coulomb barrier $^{[1,4]}$. Such simulation studies motivated a number of experiments consisting of the observation of $\alpha$-particle spectra in heavy-ion fusion-evaporation reactions in a singles mode or in coincidence with evaporation residues $^{[3,4]}$. The inability to reproduce the sub-barrier part of the observed $\alpha$-spectra, with statistical model calculations assuming spherical emission shapes, was then used as an indicator of a deformation effect.

Besides the interesting problem of selecting the proper parameter set for calculating the $T_1$'s $^{[1,4,5]}$ and therefore interpreting such data on a solid background, one must realize the following experimental facts: (a) A correlation of interest is that between the $\alpha$-emission direction and the spin direction which, in the case of fusion of spinless nuclei, is uniformly distributed on a plane perpendicular to the beam axis. (b) The $\alpha$-decay competition is normally expected to vary with the compound nucleus angular momentum. The desire for an unconstrained experimental study of these effects led to the development of the spin alignment method with the $4\pi$ $\gamma$-ray spin spectrometer$^{[8,10]}$. In this method, the magnitude and orientation of the spin of the residual nuclei is deduced on an event-by-event basis. This makes possible detailed studies, such as the measurement of $\alpha$-particle angular distributions with respect to the spin direction$^{[9]}$. Furthermore, the $\gamma$-multiplicity selection with the spin spectrometer allows us to study these decay characteristics as a function of the evaporation residue spin, which is closely correlated to the compound nucleus spin. Therefore, the alpha-decay properties of different compound nuclear systems can be studied in detail.

In this contribution, $\alpha$-particle angular distributions with respect to the spin direction are reported for a number of compound nuclei systems ranging from the closed shell $^{110}$Sn to the rare earth $^{170}$Yb. Differences in the emission patterns suggest nearly spherical and deformed emission shapes in these two extreme cases, respectively. These findings are corroborated by data of giant resonances built
on excited states of similar compound nuclear systems. This fact establishes the validity of the method as a probe of the nuclear shapes at high excitation energy and angular momentum.

2. EXPERIMENTAL METHODS

The experiments in this work were performed at the Oak Ridge heavy-ion facility (HHIRF). The compound systems studied were $^{114}$Sn, $^{138}$Nd and $^{164}$Yb[12]. Other systems studied earlier were $^{110}$Sn and $^{170}$Yb[6,7]. The reactions used to produce these compound systems are summarized in Table I.

Table I. Summary of reaction parameters of the compound systems studied.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_L^a$ (MeV)</th>
<th>Target thickness (µg/cm²)</th>
<th>$E^*^a$ (MeV)</th>
<th>$E_{CM}/V_b^b$</th>
<th>$l_{gr}^c$ (ℏ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{45}$Sc + $^{65}$Cu $\rightarrow$ $^{110}$Sn*</td>
<td>200.0</td>
<td>320-450</td>
<td>93.9</td>
<td>1.41</td>
<td>70</td>
</tr>
<tr>
<td>$^{64}$Ni + $^{50}$Ti $\rightarrow$ $^{114}$Sn*</td>
<td>250.0</td>
<td>500</td>
<td>79.5</td>
<td>1.30</td>
<td>65</td>
</tr>
<tr>
<td>$^{64}$Ni + $^{74}$Ge $\rightarrow$ $^{138}$Nd*</td>
<td>270.0</td>
<td>760</td>
<td>82.4</td>
<td>1.25</td>
<td>79</td>
</tr>
<tr>
<td>$^{64}$Ni + $^{100}$Mo $\rightarrow$ $^{164}$Yb*</td>
<td>270.0</td>
<td>990</td>
<td>67.2</td>
<td>1.14</td>
<td>74</td>
</tr>
<tr>
<td>$^{20}$Ne + $^{150}$Nd $\rightarrow$ $^{170}$Yb*</td>
<td>176.6</td>
<td>1100</td>
<td>134.8</td>
<td>2.01</td>
<td>80</td>
</tr>
</tbody>
</table>

$^a$ $E_L$ and $E^*$: beam and initial excitation energy, respectively.

$^b$ $E_{CM}/V_b$: ratio of the center of mass energy to the entrance channel Coulomb barrier $V_b = e^2 Z_1 Z_2/r_o (A_1^{1/3} + A_2^{1/3})$ where $r_o = 1.4$ fm.

$^c$ $l_{gr}$: grazing angular momentum corresponding to $\sigma_{fus} = \pi \lambda^2 \sum (2l + 1)T_l$ with $T_l = [1 + \exp((l - l_{gr})/\Delta)]^{-1}$ and $\Delta = 2$. Here $\sigma_{fus}$ was calculated with the Bass model.

Self-supporting targets of high isotopic enrichment in each of the isotopes were used (see Table I). The $\alpha$-particles were detected in five Si surface barrier telescopes positioned at the laboratory angles of 70°, 65°, 60° corresponding to ~ 90° in the center-of-mass system for the $^{114}$Sn*, $^{138}$Nd* and $^{164}$Yb* systems and in one telescope located at 160° in the laboratory. In the earlier experiments ($^{170}$Yb and $^{110}$Sn) the $\alpha$ particles were also detected near 90° in the center of mass [6,7]. The $\Delta E$ detectors had thickness of 65 µm and an acceptance cone of ~ 6° half angle. The E detectors were 1500 µm thick and served as the triggers of the spin spectrometer. The spin spectrometer served as the $\gamma$-ray detector and measured simultaneously the $\gamma$-ray multiplicity, $M_\gamma$, the total $\gamma$-ray deexcitation energy and the $\gamma$-ray angular correlations. In these experiments 71 of the 72 detectors of the spin spectrometer were used covering 95.8% of $4\pi$ sr.

The ability of the spin spectrometer in selecting rather narrow regions of spin in order to study decay patterns has been discussed earlier [8]. Since the dependence
of the $\alpha$ emission patterns vary smoothly with spin, selection of somewhat broader spin regions corresponding to coincidence fold intervals of 3 or 4 units is sufficient in these studies and helps to improve the statistics. A typical distribution of $\gamma$-ray coincidence fold ($k_\gamma$) for events triggered by $\alpha$ particles (detected at 90° in C.M.) from the $^{110}$Sn* compound system is shown in Fig. 1(a). This distribution is closely related to the $M_\gamma$ distribution and indicates that the spin distribution extends to high angular momenta for this system.

Fig. 1 (a) Distribution of the $\gamma$-ray coincidence fold $k_\gamma$ for events triggered by $\alpha$ particles detected near $\theta_{C.M.} = 90^\circ$ from 200 MeV $^{45}$Sc on $^{65}$Cu. (b) Angular distribution of the $\gamma$ rays emitted in the plane perpendicular to the beam from events with an $\alpha$ particle emitted at $\phi = 150^\circ$ (vertical arrow). The pattern is consistent with a preponderance of stretched E2 transitions. The solid curve is a least squares fit to $A_0[1 + A_2P_2(cos\phi)]$.

The method used for determining the spin direction is based on the emission of $\gamma$ radiation with a particular angular relationship to the spin direction. The $\gamma$-cascades from rotational nuclei formed in heavy-ion fusion-evaporation reactions have a preponderence of stretched E2 transitions which exhibit a doughnut-like pattern about the spin axis [$W(\theta) = \frac{5}{4}(1 - cos^4 \theta)$]. This pattern is enhanced by the presence of non-stretched dipole transitions [$W(\theta) = \frac{3}{2}(1 - cos^2 \theta)$], while the presence of stretched dipoles [$W(\theta) = \frac{3}{4}(1 + cos^2 \theta)$] has the effect of filling the hole of the doughnut. The spin direction is identified with the short symmetry axis of this pattern. This is close to the compound nucleus spin, i.e. perpendicular to the beam, provided that the misalignment caused by particle emission is small. This assumption improves at high spin where stretched particle emission dominates.
The $\gamma$-pattern for each event is projected on a plane perpendicular to the beam direction and centroid-searching methods are used to determine the angle between the sort symmetry axis and the direction of the emitted $\alpha$-particle. The use of the $\gamma$-radiation patterns in determining the spin direction can be tested by examining the $\alpha - \gamma$ angular correlations. In Fig. 1(b) we show the distribution of $\gamma$-rays in the plane perpendicular to the beam direction. Here the $\alpha$ particles were detected at an azimuthal angle $\phi = 150^\circ$ and $k_\gamma$ was between 14 and 23. This pattern is indeed consistent with a preponderance of stretched $E2$ and/or nonstretched ($\Delta I=0$) dipole transitions and justifies the use of the spin-alignment technique for obtaining the $\alpha$ angular distributions relative to the spin direction.

The factors that determine the spin orientation event by event from spin spectrometer data have been outlined in detail in our earlier work $^6$. These considerations lead to spin alignment response functions for the spin spectrometer. The two most important factors which influence these response functions are the $\gamma$-ray multiplicity and the multipolarities present in the ensemble of $\gamma$ cascades $^9$. Of these, the former is easily accounted for by simulations $^9$ while the latter may play an important role when different nuclear systems are compared.

3. RESULTS

Spectra of $\alpha$ particles were measured near $\theta_{\text{C.M.}} = 90^\circ$ for five $k_\gamma$ gates in the

![Graphs showing data for various $k_\gamma$ gates and examples of $\alpha$-particle spectra from $^{170}$Yb$^*$ corresponding to the angles of $\beta$=11$^\circ$ (closed circles) and $\beta$=81$^\circ$ (open circles) with respect to the estimated spin direction. The solid lines guide the eye. The experimental and calculated spectra integrated over $k_\gamma$ and $\beta$ are shown in the lower part by the solid squares and the dashed curve, respectively.](image-url)
range 9 to 33. Examples of such spectra are shown in Fig. 2 from the decay of $^{110}$Sn* and $^{170}$Yb* compound systems. It is seen in Fig. 2(a) that the shapes of these spectra do not change drastically with fold $k_γ$, but small differences can be seen under close examination. Fig. 2(b) shows examples of $α$ energy spectra observed at the indicated angles of $11^\circ$ and $81^\circ$ with respect to the estimated spin direction from a selected $k_γ=23-26$ gate. Differences between the spectra from the two angles with respect to the spin direction are seen near and below the evaporation Coulomb barrier. These can best be demonstrated by constructing the angular correlation between the direction of emission of $α$ particles of a given center-of-mass energy and the estimated spin direction. Examples of such angular correlations from the $^{110}$Sn* system are shown in Fig. 3 for a given $k_γ$ gate of (9–13) and two $α$ energies of 13 and 20 MeV (squares and circles) and for a given $α$ energy of 13 MeV and two $k$ gates of 14–18 and 19–23 (triangles and diamonds).

Fig. 3 Selected experimental angular correlations of $α$ particles with respect to the spin direction of the residual nucleus. The triangles and diamonds correspond to $E_α = 13$ MeV for the $k_γ$ gates of 14–18 and 19–23, respectively. The squares and circles correspond to the $k_γ$ gate of 9–13 for $E_α = 13$ and 20 MeV, respectively. The solid lines are least-squares fits of $A_0[1 + A_2P_2(cos(\beta))]$ to the data.
respectively). We see that for a given $E_\alpha = 13$ MeV the anisotropy of the correlation increases as $k_\gamma$ is increased, indicating a favoring of $\alpha$ emission perpendicular to the spin direction. We also observe that, for a given $k_\gamma$ gate, the emission perpendicular to the spin direction also increases with increasing $E_\alpha$.

The dependence on $E_\alpha$ of the measured correlations from all the systems can be best demonstrated by fitting the correlations to the function $W(\beta) = A_0[1 + A_2P_2(cos\beta) + A_4P_4(cos\beta)]$. The $A_4$ coefficients have, in general, been found to be small and essentially independent of $E_\alpha$. We therefore report the $A_2$ coefficients as a function of $E_\alpha$. Examples of fits with the above expression are shown as solid lines in Fig. 3. It is clear that a negative $A_2$ value indicates an anisotropy of $W(90^\circ)/W(0^\circ) > 1$. The extracted $A_2$ coefficients are plotted vs. $E_\alpha$ in Fig. 4 for various $k_\gamma$ gates, as indicated, for the $^{110}$Sn$^*$, $^{170}$Yb$^*$ and the $^{114}$Sn$^*$, $^{138}$Nd$^*$, $^{164}$Yb$^*$ systems, respectively. The Coulomb barriers for each system in Fig. 4 are indicated by the vertical arrows. Certain remarkable trends are observed in these results. For each of the systems studied, for a given $E_\alpha$ the $A_2$ values become monotonically more negative as the coincidence fold $k_\gamma$ is increased. The dependence of $A_2$ on $E_\alpha$, however, exhibits patterns characteristic of each system under study. Thus, for the $^{110}$Sn$^*$ and $^{114}$Sn$^*$ systems the $A_2$ coefficients become monotonically more negative (larger anisotropies) as $E_\alpha$ is increased from values well below the barrier to the highest energies where measurements were possible. In contrast, $^{170}$Yb$^*$ shows $A_2$ coefficients which have an absolute maximum at the barrier and become more negative at lower and higher $E_\alpha$ values. An intermediate behaviour is observed for the other systems $^{114}$Sn$^*$, $^{138}$Nd$^*$ and $^{164}$Yb$^*$.

4. DISCUSSION

The results presented in the previous section on the spin and $E_\alpha$ dependence of the angular distribution coefficients $A_2$ relative to the spin direction, indicate behavior patterns characteristic of the various decaying compound systems. In order to understand these patterns, detailed statistical model simulations are needed to account for all the possible decay paths of the compound nuclei involved.

The $\alpha$-particle angular distribution for an initial spin $I_i$ and final spin $I_f$ relative to the $I_f$ direction is given by:

$$W_{E_\alpha, I_i, I_f} (\beta) = \sum_\lambda a_{E_\alpha, I_i, I_f, \lambda} B_{\lambda}(I_i) P_{\lambda}(cos\beta)$$  \hspace{1cm} (1)

where

$$a_{E_\alpha, I_i, I_f, \lambda} = \frac{T_l(E_\alpha)}{\sum_\nu \frac{T_{\nu}}{T_{\nu}(E_\alpha)}} (-1)^{I_i+I_f}(2l+1)(2I_i+1)^{1/2}(2\lambda+1)^{1/2}$$

$$\times \left\{ \begin{array}{ccc} l & l & \lambda \\ 0 & 0 & 0 \end{array} \right\} \left\{ \begin{array}{ccc} l & l & \lambda \\ I_i & I_i & I_f \end{array} \right\}$$  \hspace{1cm} (2)
The $T_l(E_\alpha)$ are transmission coefficients and $B_\lambda(I_i)$ are the statistical tensors describing the ensemble of spin orientations with respect to the quantization axis. The $B_\lambda(I_i)$ are derived from the spin alignment responses of the spin spectrometer using the vector model as described in Ref.6. In order to calculate the $A_2$ for comparison with experiment the events from the statistical model calculations using the code JULIAN-PACE$^{[18]}$ are sorted for given gates on $E_\alpha$ and $M_\gamma$. The corresponding distributions in $\Delta I = I_i - I_f$ are obtained and for each $\Delta I$ bin the respective $< I_i >$ are derived. From the $\Delta I$ distributions the relative weights for each $l$-value are obtained from the triangular condition $|\Delta I| \leq l \leq 2 < I_i > - \Delta I$. The level density parameter, in the $^{170}$Yb case, was taken as $A/9.5$ and the Sierk$^{14}$ yrast lines were used. The optical model parameters of Huizenga and Igo$^{[16]}$ for $\alpha$-emission were used. Such a choice of parameters reproduces the experimentally measured cross sections$^{[15]}$.

The calculated $A_2$ coefficients, for $^{170}$Yb$^*$, agree well with the monotonic decrease of the experimental $A_2$ values above the Coulomb barrier ($\sim$20 MeV for a spherical nucleus), but do not reproduce the decrease of $A_2$ at low $E_\alpha$ (Fig. 4(b)).

The fact that the observed deviation occurs at emission energies sensitive to barrier penetration effects, has suggested the nuclear deformation as a possible factor for the decrease of the measured $A_2$ coefficients at low $E_\alpha$. If the emitting system is deformed with its longest axis perpendicular to the spin direction, the subbarrier $\alpha$-particles will be emitted preferentially along this direction (because of the lower Coulomb barrier). This leads to decreasing $A_2$ coefficients with decreasing $E_\alpha$. On the other hand, the $\alpha$-particles above the barrier would not be affected much by the deformation since their emission is mainly determined by the level densities. Therefore, the observed deviation can be interpreted as a deformation effect which increases with spin.

In contrast to this finding, the trend of the $A_2$ coefficients calculated above is consistent with the observed emission pattern for the $^{110}$Sn$^*$ and $^{114}$Sn$^*$ compound nuclei. A similar statistical model calculation for $^{114}$Sn$^*$ is compared with the experimental data in Fig. 4(c) and shows agreement. The behaviour of these two systems is just that predicted for a spherical emitting nucleus.

The implied differences in the shapes of the Yb$^*$ and Sn$^*$ compound nuclei are supported by the observation of the decay of giant dipole resonances built on excited states of similar compound systems. Giant resonance data from the decay of $^{166}$Er$^*$ (61.5 MeV) suggest a two-component resonance in contrast to the decay of $^{108}$Sn$^*$ (61.2 MeV) where a single resonance peak was observed$^{[11]}$. This fact establishes the validity of our method as a probe of the nuclear shapes at high excitation energy and angular momentum.

Comparing with the behaviour of the experimental $A_2$ coefficients for the other systems of Fig. 4, we observe a mild deformation effect in the decay of $^{138}$Nd$^*$. A stronger deformation is implied by the $^{164}$Yb$^*$ data which is weaker than the one
Fig. 4 $A_2$ coefficients as a function of $E_{\alpha,CM}$ from the five systems studied. (a) Results from $^{110}$Sn* decay. The arrows indicate the position of the spherical barrier calculated with $r_0 = 1.4$ fm. The squares, circles and triangles are for $k_\gamma$ bins of 9-13, 14-18 and 19-23, respectively. The dashed line guide the eye. (b) Results from $^{170}$Yb decay. From top to bottom, the closed circles, open circles, triangles, large closed circles and closed squares correspond to $k_\gamma$ bins of 11-14, 15-18, 19-22, 23-26 and 27-32 (i.e. 34, 43, 51, 59 and 64), respectively. The pairs of curves are FWHM boundaries of the $A_2$ coefficients from a statistical model calculation using transmission coefficients from a spherical optical model potential. In (c), (d) and (e) the results for $^{114}$Sn*, $^{138}$Nd* and $^{184}$Yb* are shown; The open squares, circles, closed triangles (shifted by -0.2), diamonds (shifted by -0.4) and closed squares (shifted by -0.6) correspond to the $k_\gamma$ bins of 11-14, 15-18, 19-22, 23-26 and 27-33, respectively.
observed in its neighbouring $^{170}$Yb* system.

A number of statistical model calculations are in progress for a quantitative understanding of the behaviour of these systems. The role of $\gamma$-multiplicity in selecting high spin $\alpha$-decays is depicted in Fig. 5 where the pre-evaporation states involving $\alpha$-decay are shown as a function of the compound nucleus angular momentum for different multiplicity bins. Fig. 5(a) and (b) show the result of a PACE calculation for the decay of $^{114}$Sn* and $^{170}$Yb*, respectively. Both figures show that high multiplicity bins select states which originate from high compound nucleus angular momentum. It is also clear, that the angular momentum selectivity, via $\gamma$-ray multiplicity, improves for the heavier and the more neutron-rich the compound nuclei.

**Fig.5** (a) Pre-evaporation distribution of the states which involve at least one $\alpha$-decay as a function of the $^{114}$Sn* compound nucleus spin. The total, and the $M_\gamma$-multiplicity gated distributions in the bins 6-10, 11-15, 16-20, 21-25 and 26-30 are shown. (b) The same as in (a) for $^{170}$Yb*.

5. CONCLUSIONS

Summarising, the spin-alignment method with the spin spectrometer has provided us with a unique method of observing $\alpha$-particle angular distributions with respect to the spin direction of residual nuclei on an event-by-event basis. Such measurements have been shown to be sensitive on the shape of the emitting systems at high excitation and angular momentum.

A survey study of the $\alpha$-decay properties of different compound nuclear systems shows quantitative differences which can be attributed to the shape of the emitting nucleus. The extracted anisotropy coefficients suggest emission from spherical shapes in the case of the Sn* isotopes. Small deformation effects are suggested by the $^{138}$Nd and $^{164}$Yb data. The $^{170}$Yb data indicate the largest deformation that increases considerably with spin. These results are in agreement with similar findings for systems in which the decay of the giant resonances built on excited
states has been studied. This observation strengthens the belief that alpha-particle emission can be used as an effective probe of the nuclear shapes at high excitation energy and angular momentum.

REFERENCES