Superdeformed bands in $^{80}\text{Sr}$ and the evolution of deformation in Sr isotopes


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Four superdeformed bands are reported in $^{80}\text{Sr}$, extending known superdeformation in the $^{80}\text{Sr}$ series down to $N = 42$. The characteristics of these bands are discussed. Residual Doppler shifts were measured and average transition quadrupole moments ($Q_t$) inferred for these new bands. These $Q_t$ values are compared to $Q_t$ values obtained for previously identified superdeformed bands in $^{81-83}\text{Sr}$. The low $Q_t$ of 2.7 $^{+0.7}_{-0.6}$ e bent obtained for the yrast band in $^{80}\text{Sr}$ indicates a reduction in the deformation of yrast superdeformed bands in the series $^{80-84}\text{Sr}$ with decreasing $N$, and possibly the onset of triaxiality in superdeformed shapes.

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Recent experimental advances have led to the identification and study of superdeformed (SD) states in nuclei in a $A \sim 80$ region. The first such observation was made by Baktash et al. in $^{83}\text{Sr}$ [1]; subsequent residual Doppler shift measurements established that this band is in fact highly deformed [2]. Rapid progress has been made in this area, due in large part to the use of charged-particle channel selection devices [3] coupled to modern high efficiency Ge detector arrays [4-6]. As a result, SD bands have now been established in $^{81,83}\text{Sr}$ [7-9], $^{82,84}\text{Y}$ [10,11], $^{83,84}\text{Zr}$ [12,13], $^{86}\text{Zr}$ [14] and $^{87}\text{Nb}$ [15]. Multiple bands are known in many of these nuclei, and many of these bands have been shown to be highly deformed by lifetime measurements.

A detailed understanding of the structure of SD bands in this mass region is still emerging. Most of the SD bands in this region have been interpreted as prolate shapes with $\beta \approx 0.5$, due to their highly collective nature, large moments of inertia, and in some cases measured average transition quadrupole moments [2,9,13,12]. However, it has been suggested [16-18] that a large variety of shapes is possible in the high and intermediate spin range in these nuclei, including both prolate and triaxial collective structures. Triaxial SD shapes have also been suggested theoretically [19] in the $A \sim 90$ region. A recent report suggests triaxial ($\gamma \sim 20^\circ$) SD structures in $^{86}\text{Zr}$ [14]. This is somewhat in contrast to the situation for superdeformation in heavier mass regions, in which the only reported triaxial SD bands are in $^{153,163,169}\text{Lu}$ [20].

With these issues in mind, a study of the evolution of deformation in the SD bands of $^{81,83}\text{Sr}$ isotopes was undertaken. In addition, the Sr isotopes offer the opportunity to study the role of neutron excitations in the superdeformed structures of at least three adjacent isotopes. Of particular interest would be the establishment of SD bands in $^{82}\text{Sr}$, since this would extend the SD sequence in Sr isotopes down to the point at which only one neutron excitation of the $h_{11/2}$ orbital should be yrast, perhaps leading to a drop in the deformation of the SD states. A new experiment was conducted to study high-spin states in $^{81,82}\text{Sr}$. The results on $^{82}\text{Sr}$ [9] and the extensive data on normal deformed states in $^{83}\text{Sr}$ [21] will be presented elsewhere. The data reported in references [2,7] were reanalyzed to extract transition quadrupole moments ($Q_t$) for the SD bands in $^{81,83}\text{Sr}$. These results and the $^{83}\text{Sr}$ SD search results are reported here.

The experiment was performed at the 88-Inch Cyclotron at the E. O. Lawrence Berkeley National Laboratory using the GAMMASPHERE array [6] then consisting of 57 Compton-suppressed Ge detectors, and the $4\pi$ charged-particle detector array MICROBALL, which consists of 95 CsI(Tl) scintillators with photodiode read-out [3]. A $^{58}\text{Ni}$ beam at an energy of $130$ MeV incident on a 99.7% enriched, 330 $\mu g/cm^2$ self-supporting $^{58}\text{Ni}$ target was used to populate high spin states in $^{80}\text{Sr}$ via the $\alpha+2p$ fusion-evaporation channel. The typical beam intensity was 5 pA, and the event trigger required three or more simultaneous Compton-suppressed Ge detector signals. In four days of beam time $1.3 \times 10^5$ events were recorded.

Proton and alpha particles were identified and cleanly separated using two coupled pulse-shape discrimination techniques [3]. The MICROBALL particle detection and
identification efficiencies were in excess of 80% for a proton and 65% for an α particle in this analysis. The reduced efficiency for the α particles is mainly due to stopping in the absorbers in front of the MICROBALL detectors at back angles. In addition to exit-channel selection, the charged particles detected in the MICROBALL were used to determine the moments of the recoiling residual nuclei on an event-by-event basis. This allowed a more precise Doppler-shift correction to be made, improving the γ-ray energy resolution by a factor of approximately 2.5 [3]. The α2p gate contained small admixtures of higher charged-particle fields (primarily α3p), and of mis-identified channels (for example, the 3p channel in which one proton is mis-identified as an α particle). These were, however, minor components to the α2p gated events, which contained ≈ 70% 88Sr, as well as smaller amounts of 78,79Sr (the α2p2n and α2p4n channels, respectively). The α2p-gated events were sorted off-line into various Eγ - Eγ matrices and Eγ - Eγ - Eγ cubes, and analyzed using the Radware software package [22].

The data were searched for SD structures, and four SD bands were found. These bands are shown in Fig. 1, in which the spectra were generated by double-gating on all non-redundant combinations of the labelled transitions. All four bands are assigned to 88Sr on the basis of their satisfying the α2p charged-particle gate, and the observation of coincidences with low spin transitions in 86Sr [21,23]. In Figs. 1 and 2 SD band 3 (SD3) is seen to feed into SD band 2 (SD2), as shown by the SD2 transitions (marked by asterisks) in the SD3 spectrum; however, no linking interband transitions could be identified. In addition, no transitions between the SD bands and the low-spin level scheme of 86Sr were seen. The SD sequences are shown in Fig. 2. Note that SD2 “forks” at the top of the band, and the assignment of which transitions are “in-band” is arbitrary. Spin estimates are made based on the observed feeding into known levels of 86Sr, assuming ≈ 25% in unseen transitions between the SD and lower spin states. Directional correlation (DCO) [24] ratios, obtained using single gated data, confirmed that all of the transitions are quadrupole in nature. The dynamic moments of inertia (J(2)) of the SD bands are shown in Fig. 3; these are seen to be typical of SD bands in this mass region. These SD bands in total are populated with ≈ 3.5% of the 86Sr yield in this reaction.

Average lifetimes of the states in the SD bands were measured using the Doppler shift attenuation technique [25], in which the slowing of the recoil ion in the thin target is used as a clock to determine the γ-decay lifetimes of the fastest transitions. Fig. 4(a) and (b) show as an example the resulting residual Doppler shift for some transitions in SD1. F(τ) = β/β0 for data for bands SD2, SD3 and SD4 in shown in Fig. 4(c); Fig. 4(d) shows the F(τ) data for SD1 and some non-shifted normal deformed (ND) transitions. Fits are made to the F(τ) function in order to determine the transition quadrupole moment (QT), using the model described in Refs. [13,25]. Side feeding is included based on the measured intensity pattern, and the side feeding times are assumed to be less than those of the preceding in-band transitions. A sample fit (solid line) is shown in the bottom panel of Fig. 4 for SD1; the dashed lines represent the quoted uncertainty.

The results of the QT fits for all four bands in 88Sr are summarized in Table I. All of the SD bands are significantly deformed, with QT values between 2.2 and 3.6 eb, and the data suggest that there is some variation in the deformations for these bands. In order to interpret these results, a study of QT values in the Sr series was also conducted. For this study, the data in references [2,7] were reanalyzed in order to extract F(τ) measurements for the SD bands in 81,82Sr, respectively. The results of this reanalysis are also shown in Table I. In all cases the errors quoted for QT measurements include the statistical uncertainty associated with the measurement and an estimate of the additional error due to the uncertainty in the spin assignments (typically ± 2h). Systematic errors involved in the model used to fit the F(τ) data are not included, though variations in the fits using different side-feeding times are smaller than the quoted errors. Systematic errors are difficult to estimate, but are thought to be on the order of 20% in QT.

The evolution of transition quadrupole moments in the yrast SD bands of Sr isotopes with N = 42 to 45 is shown in Fig. 5(b). Also shown are the corresponding (average) QT values for low spin states in the ground bands of the same nuclei, taken from Ref. [26]. A distinct trend to lower QT as N decreases from 44 to 42 is seen in the data, so that the apparent deformation of the yrast SD band in 84Sr is comparable to the deformation at low spin of the 84Sr ground band. (The QT trend shown in Fig. 5(b) for ND states reflects configuration changes in the yrast structures at low spin in these nuclei.)

The low values of QT for most of the SD bands in 86Sr immediately raises the concern that these bands are not “superdeformed” at all. However, they possess all of the properties of SD bands in this mass region, such as the typical SD feeding and decay out pattern, the lack of direct linking γ-ray transitions to the rest of the level scheme, a demonstrably high-spin nature, and dynamic moments of inertia that are relatively large (≈ 25h2/MeV) and constant. The population of either neutron or proton h1/2 excitations is presumed to be responsible for all of the SD bands in the A ~ 80 region [16,17], and the above assignment is therefore consistent with these bands in 86Sr being “superdeformed.” This definition of superdeformation in this mass region may be debatable, since high-deformation structures at high spin which do not involve h1/2 occupations do exist in this mass region [27] (and the low-spin structures can be highly deformed), but it is the definition used here.

A comparison of the yrast bands in 86Sr and 88Sr is perhaps useful in this connection. Both yrast SD bands show an upbend at their highest spins, as can be seen in Fig. 5(a). The similarity of the J(2) values of these two bands is suggestive of similar configurations being
involved. Since the yrast SD band in $^{81}$Sr has been interpreted to be an excitation of one $h_{11/2}$ neutron and no $h_{11/2}$ protons - $\pi h_2^1 \pi h_0^0$ (below the crossing) [7], the same assignment is given to the yrast band in $^{85}$Sr.

Previously published configuration-dependent shell-correction calculations using a Woods-Saxon potential [16] of the lowest energy configurations in $^{85}$Sr in fact predict that, in the spin range (35 - 45 $\hbar$) in which these SD bands are populated, moderately triaxial shapes are expected. Table I lists the assignments of the four SD bands, with the $\beta$ and $\gamma$ values given for these bands in Ref. [16]. These assignments are based on the observed properties of the bands, coupled with the expectation that the lowest energy configurations in the relevant spin range would be populated. The table also compares the observed $Q_i$ values for these bands with those inferred from the shape parameters predicted by the calculation. These are obtained from the usual relation between $Q_i$ and $(\beta, \gamma)$ [17]:

$$Q_i^{th} \approx Q_i^{th}(\beta, \gamma = 0) \cdot \frac{2}{\sqrt{3}} \cdot \cos(\gamma + 30^\circ),$$

and $Q_i^{th}(\gamma = 0) = \frac{3}{\sqrt{2\pi}} f(\beta) Z r_0 A^{1/3} \beta^2$, with $r_0 = 1.20$ fm and $f(\beta) \approx (1 + 0.36 \beta + 0.1023 \beta^2 - 0.0352 \beta^3)$. In the cases discussed here, in which $\gamma$ is positive, the measured $Q_i$ values are indicative of larger deformation than in the case of $\gamma = 0$. For a $\gamma$ of $10^\circ$, the deformation parameter $\beta_3$ is $13\%$ larger than that with $\gamma = 0$, for the same value of $Q_i$.

The assignment for the yrast SD band (SD1) based on the calculations in Ref. [16] is, as suggested above, the same configuration as the yrast band in $^{81}$Sr. SD1 exhibits the beginning of a crossing at $\hbar \omega \approx 1.3$ MeV, slightly higher than the predicted value of $1.2$ MeV. Above this crossing, occupation of the first $h_{11/2}$ proton orbital is expected. SD2 is similarly assigned, with the added observation that its measured $Q_i$ appears slightly higher than the other bands. It should be noted, however, that SD2 appears to have a crossing at $\approx 1.0$ MeV, a feature which is not expected for the configuration to which it is assigned. Since bands SD3 and SD4 are populated at somewhat higher spin than SD1 and SD2, they are assigned to configurations which become considerably less yrast below $\approx 35 \hbar$. This behavior is consistent with the observed intensity patterns, and with the observation mentioned earlier that SD3 feeds into SD2. However, since the calculation of the configuration energies is subject to some uncertainty, and little additional experimental information (such as alignments) is available, the specific assignments for bands SD3 and SD4 are speculative. All of the observed SD bands in $^{85}$Sr, however, can be interpreted qualitatively as deformed, triaxial structures, based on their observed transition quadrupole moments and large dynamic moments of inertia.

Table I lists the assignments given in the literature for $^{81,85}$Sr and the resulting $Q_i^{th}$ values for the assumed values of $\beta$ and $\gamma$ [7,2]; the same theoretical $Q_i^{th}$ data are shown in Figure 5(b) for the yrast SD bands in $^{80-85}$Sr as a dot-dash line. $^{82}$Sr is expected to have a more stable prolate SD configuration, due to the $N = 44$ high-spin shell closure, and the SD minimum becomes more triaxial as $N$ decreases. The SD band in $^{83}$Sr is expected to be slightly more deformed than that in $^{82}$Sr, due to the population of an additional $\nu h_{11/2}$ orbital. This assignment is consistent with the higher $J^{(\pi)}$ value in $^{85}$Sr, as seen Fig. 5(a). However, the $Q_i$ value for $^{83}$Sr presented here does not appear to agree with this assignment. Given the possibility of a considerable systematic component to the experimental errors in $Q_i$, no definite resolution of this discrepancy can be made with the current data. Detailed calculations of the SD shapes in all the Sr isotopes would help clarify the nature of the observed variation in measured $Q_i$ values over this sequence.

In summary, four new SD bands were identified in $^{81}$Sr. Their properties, including measured average transition quadrupole moments, are compared to theoretical predictions. Very reasonable agreement is found, suggesting that these bands have deformations $\beta \approx 0.4$ and $\gamma \approx 5 - 20^\circ$. The results indicate that along the Sr series the deformation ($\beta_2$) of SD structures drops as $N$ decreases from $N = 44$ to 42, consistent with calculations which also indicate that triaxiality becomes important in $^{85}$Sr. New $Q_i$ results are presented for $^{81,85}$Sr, and the trend in deformation along the Sr series is consistent with expectations, with the exception of the SD band in $^{85}$Sr, for which the experimental $Q_i$ value is too low. Further theoretical and experimental work directed to understanding the evolution of SD shapes in this region is needed.

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[14] D. G. Sarantites et al., to be published.
[21] D. Winchell et al., to be published.

FIG. 1. The four SD bands observed in $^{80}$Sr, obtained by double-gating on the labelled transitions. Asterisks show SD2 lines visible in the SD3 spectrum, indicating feeding of SD3 into lower spin states in SD2.

FIG. 2. Partial level scheme of $^{80}$Sr, showing the four new SD bands.

FIG. 3. Dynamic moments of inertia for the four SD bands in $^{80}$Sr.

FIG. 4. $\gamma$-ray energy spectra single-gated on transitions in SD1 for (a) front (low $\theta$) and (b) back Ge detectors, showing the residual Doppler shifts. No residual shift would correspond to $\gamma$ decay outside the target. F$(\tau)$ plots for transitions in SD1 (d) and SD2, SD3, and SD4 (c). A fit with $Q_{\beta} = 2.7 \pm 0.3$ eb is shown for SD1, as are the data for unshifted normal deformed (ND) transitions.

FIG. 5. Dynamic moments of inertia ($J_z^{(2)}$) for the yrast SD bands in $^{80-83}$Sr (a). The evolution of the average transition quadrupole moment ($Q_{\beta}$) for the yrast SD bands in $^{80-83}$Sr (b). Low-spin $Q_{\beta}$ values [26] for the ground bands of the same isotopes are shown for comparison. Also shown (solid line) are the expected $Q_{\beta}$ values based on the assumed assignments and calculations in Ref. [16]. Lines connecting data points are to guide the eye.

TABLE I. Fitted $Q_{\beta}^{\text{exp}}$ values for SD bands reported for $^{80,81,83}$Sr. Quoted errors are statistical only, except that the uncertainty in spin assignments is included. SD band assignments for $^{80}$Sr are compared to their calculated [16] properties. Theoretical $Q_{\beta}^{\text{th}}$ values are obtained from the relations in the text, using theoretical $\beta$ and $\gamma$ values. The assignments for $^{81}$Sr and $^{83}$Sr are taken from refs. [7] and [2], respectively.

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<th>$\beta^{th}$</th>
<th>$\gamma^{th}$</th>
<th>$Q_{\beta}^{th}$ (eb)</th>
<th>$Q_{\beta}^{\text{exp}}$ (eb)</th>
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<tr>
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<td>2.8 $^{+1.1}_{-1.0}$</td>
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